SRI VENKATESWARA INTERNSHIP PROGRAM

FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

Project Report–2020

”Prey–Predator Model of Real Life Problem and Dynamical Behavior”

Sri Venkateswara College
University of Delhi
Dhaula Kuan
New Delhi–110021

SRIVIPRA PROJECT 2020

Title : Prey–Predator Model of Real Life Problem and Dynamical Behavior

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SRIVIPRA 2020

Signature of Mentor

SRI VENKATESWARA INTERNSHIP PROGRAM
FOR RESEARCH IN ACADEMICS
(SRI-VIPRA)

<u>Project Report -2020</u>

"APPLICATION OF COMPUTATIONAL FLUID DYNAMICS IN THE DETECTION OF GLAUCOMA"

Sri Venkateswara College

University of Delhi

Dhaula Kuan

New Delhi -110021

SRIVIPRA PROJECT 2020

Title: Application of Computational Fluid Dynamics in the Detection of Glaucoma

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SRIVIPRA 2020

Certificate

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project entitled "**Application of Computational Fluid Dynamics in the Detection of Glaucoma"**. The participants have carried out the research project work under my guidance and supervision from July 1, 2020 to July 31, 2020. The work carried out is original and carried out in an online mode.

Signature of Mentor

(Dr. Swarn Singh)

Acknowledgements

We wish to extend our heartfelt gratitude to Prof. Swarn Singh from Department of Mathematics, Sri Venkateswara College, University of Delhi for his regular guidance, without which we would not have been able to accomplish this project.

15th August, 2020 Anant Narayanan, Prassanna Nand Jha

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Abstract

Glaucoma is a group of eye diseases caused due to damage to a nerve in the back of the eye called the optic nerve. It is one of the leading causes of blindness in adults over sixty years of age with about 6 to 67 million people having glaucoma globally. There are many different types of glaucoma, but the most common type is called open-angle glaucoma. People suffering from glaucoma have frequently been diagnosed with high eye pressure (intraocular pressure) and among the several risk factors, a high intraocular pressure represents the most consistent aspect leading to open-angle glaucoma. In this paper, we study the application of partial differential equations in modelling intraocular pressure under buoyant flow due to temperature differences specific to the anterior chamber of the human eye under certain basic assumptions. To begin with, we use basic Navier - Stokes equations and further build upon them to obtain formulations for pressure and temperature using Computational Fluid Dynamics, thus determining the risk of glaucoma mathematically.[5]

Keywords:

1) Buoyancy Driven Flow: A primarily horizontal flow in a gravitational field that is driven by a density difference.

2) Trabecular Meshwork: The trabecular meshwork is an area of tissue in the eye located around the base of the cornea, near the ciliary body, and is responsible for draining the aqueous humour from the eye via the anterior chamber.

3) Anterior Chamber: The anterior chamber is the front part of the eye between the cornea and the iris.

4) Critical Pressure (p_c): The critical value of Intraocular pressure of the eye , above which the person may experience symptoms of Glaucoma. It is 21 mmHg or 2.8 kPa for a normal eye.

5) Ambient Pressure (p_a): The ambient pressure on an object is the pressure of the surrounding medium, such as a gas or liquid, in contact with the object.

1. Introduction

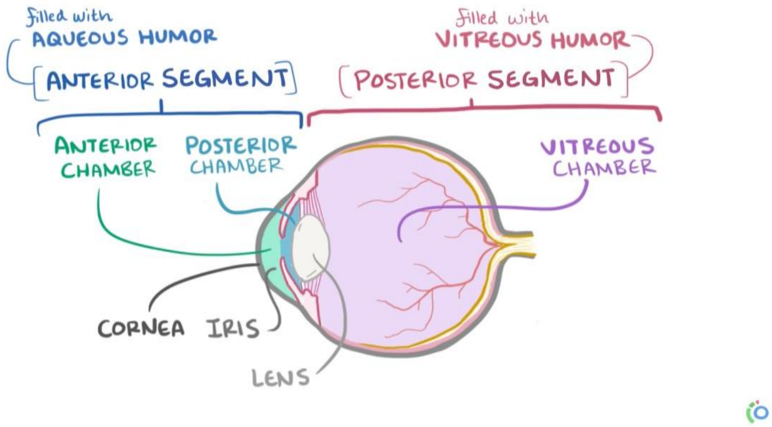
1.1 UNDERSTANDING THE ANATOMY OF THE HUMAN EYE

The human eye is an organ that reacts to light and allows vision. The eye is not shaped like a perfect sphere, rather it is a fused two-piece unit, composed of an anterior (front) segment and the posterior (back) segment. The anterior segment is made up of the cornea, iris and lens. The cornea is transparent and more curved, and is linked to the larger posterior segment. The spaces of the eye are filled with the aqueous humour anteriorly, between the cornea and lens, and the vitreous body, a jelly-like substance, behind the lens, filling the entire posterior cavity. The aqueous humour is a clear watery fluid that is contained in two areas: between the cornea and the iris, and between the iris and the lens. It is produced by the ciliary epithelium and accumulates in the posterior chamber, further flows through the pupil into the anterior chamber[3]. It then exits the anterior chamber via one of three routes:

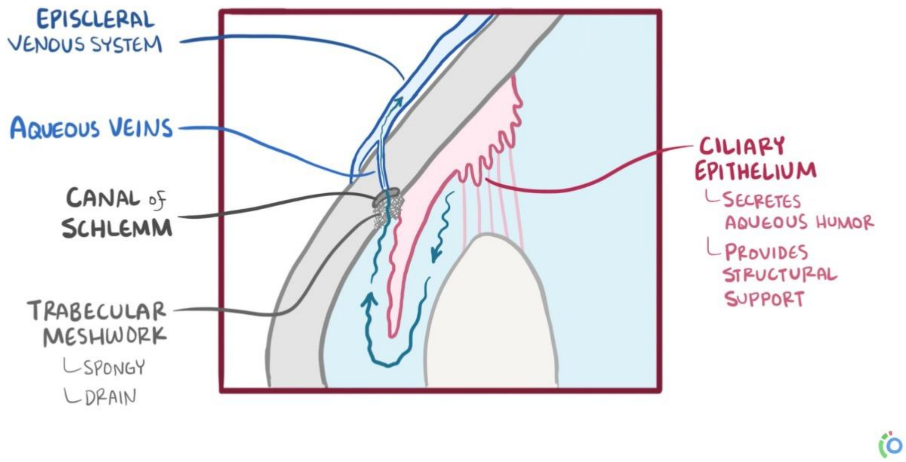
1) The vast majority of aqueous humour drains through the trabecular meshwork at the angle of the anterior chamber and into the Schlemm canal where it enters episcleral veins.

2) A small amount of the aqueous humour passes into the suprachoroidal space and enters venous circulation in the ciliary body, choroid, and sclera.

3) A still smaller amount of aqueous humour transits through the iris and back into the posterior chamber.



(a) Lateral Cross-section of human eye



(b) Zoomed-in figure of Anterior Chamber of Human Eye

These entry and exit points regulate the flow of aqueous humour regularly, thus maintaining a healthy interocular pressure (about 21 mm Hg) in the eye. This intraocular pressure (IOP) can be defined as the fluid pressure inside the eye, involving the magnitude of the force exerted by the aqueous humour on the internal surface area of the anterior eye.

In the vast majority of Glaucoma cases, the intraocular pressure (IOP) is higher than normal and therefore, the hypothesis that evaluation of IOP causes optic nerve damage and hence visual impairment is generally accepted. The elevated IOP is caused by an increased resistance to the outflow of aqueous humour from the eye either through the conventional outflow pathway or that the combined resistance of the trabecular meshwork filled with biopolymer together with the inner lining of Schlemm's canal is estimated to be sufficient to be the primary source of resistance to the outflow of aqueous humour in healthy humans. It has long been acknowledged that aqueous humour in the anterior chamber can circulate under the action of buoyancy-driven currents. The driving mechanism for these currents is the temperature difference between the anterior surface of the cornea (which, under normal waking conditions is exposed to ambient conditions) and the iris (which is maintained essentially at body temperature).This paper actively accounts for this bouyant force while obtaining the corresponding partial differential equations.[4]

1.2 WHAT IS COMPUTATIONAL FLUID DYNAMICS ?

Computational fluid dynamics (CFD) is a branch of fluid mechanics that uses numerical analysis and data structures to analyze and solve problems that involve fluid flows. Computers are used to perform the calculations required to simulate the free-stream flow of the fluid, and the interaction of the fluid (liquids and gases) with surfaces defined by boundary conditions. CFD analysis consists of three main steps: pre-processing, processing and post-processing - here is a brief introduction to each of them.

1. Pre-Processing
   Pre-processing is the first step in CFD simulation - which can help to define the parameters of the simulation in an accurate way, if done properly.

2. Processing
   Every simulation process through a CFD program has to follow a defined set of steps. After all, simulations are a set of steps that must be complied with - as these set guidelines help to avoid getting stuck or receiving error messages in subsequent stages.

3. Post-Processing
   After getting results in the simulation stage, the next step is to analyse those results. Using the available methods such as vector plots, contour plots, data curves and streamlines to achieve this, we get accurate reports and graphical representations. However, the basis for all simulation softwares and studies is the mesh, also called a grid. Mesh or grid is defined as a discrete cell or elements into which the domain/ model is divided. All the flow variables and any other variables are solved at centres of these discrete cells. This entire process of breaking up a physical domain into smaller subdomains (elements/ cells) is called mesh generation. Therefore meshing is an extremely critical part of CFD process and without understanding it, we cannot proceed to solve or even expect any relevant results. Also the accuracy of the CFD simulation results is directly linked to meshing. The better the mesh in quality, the more accurate results can be expected.

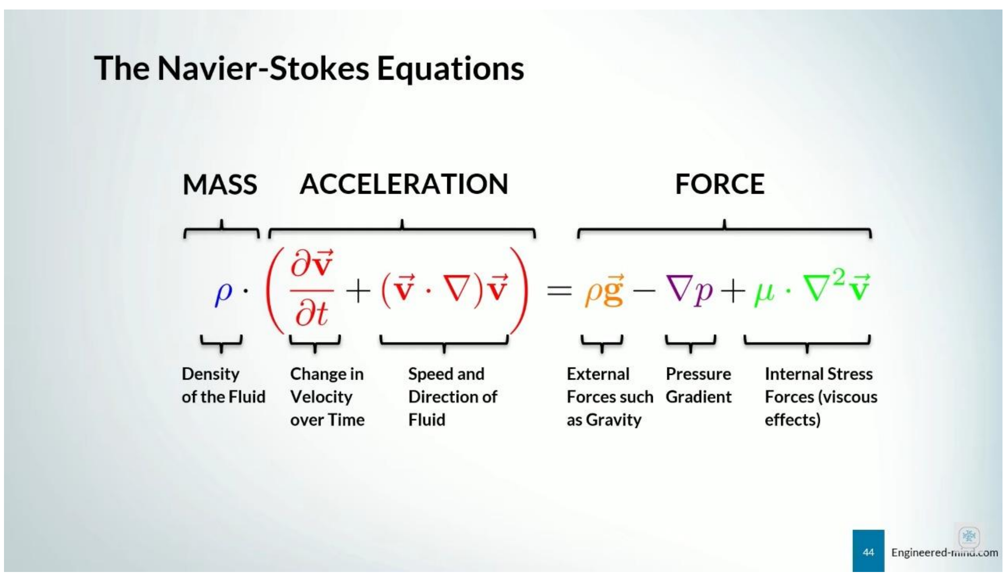
2. Goal of the study

2.1 We use basic Navier - Stokes equations and further build upon them to obtain formulations for pressure and temperature using Computational Fluid Dynamics, thus determining the risk of glaucoma mathematically.

2.2 The major effort has been put in deducing the differential equations from the very standard Navier - Stokes equations with respect to the geometry of the eye.

3. Methods

3.1 Standard Navier-Stokes Equations



The Navier-Stokes Equations are a set of differential equations which describe the motion of viscous fluid substances. This balance equation arises from applying Isaac Newton's second law to fluid motion, together with the assumption that the stress in the fluid is the sum of a diffusing viscous term (proportional to the gradient of velocity) and a pressure term—hence describing viscous flow. In this study, the "lubrication theory" limit of the Navier–Stokes equations was appropriate. Further, using the Boussinesq approximation for the buoyancy, a set of partial differential equations was formed for further analysis.

3.2 Basic Assumptions

For the purpose of modelling, we made some key assumptions, as follows:

1) There is no flow of Aqueous Humour through the pupil aperture
2) Gravity acts along the positive x-axis
3) The temperature difference between the posterior surface of cornea and the anterior surface of iris is constant

Note: Further assumptions have been mentioned as and when required

3.3 Parameters used

This is a table containing the list of parameters used all throughout this paper and their respective constant values

Symbols	Parameters	Values	Units
a	Radius of Anterior Chamber	5.5×10^{-3}	metres
l	Total width of Anterior Chamber	11×10^{-3}	metres
α	Coefficient of linear expansion of aqueous humour	3.0×10^{-4}	per kelvin
g	Acceleration due to gravity	9.8×10^{0}	metres/second^2
μ	Dynamic Viscosity of Aqueous Humour	1.0×10^{-3}	Pascal second
h_0	Height of Anterior Chamber	2.75×10^{-3}	metres
ρ_0	Density of Aqueous Humour	1.0×10^{3}	kilogram/metres^3

3.4 The Differential Equations

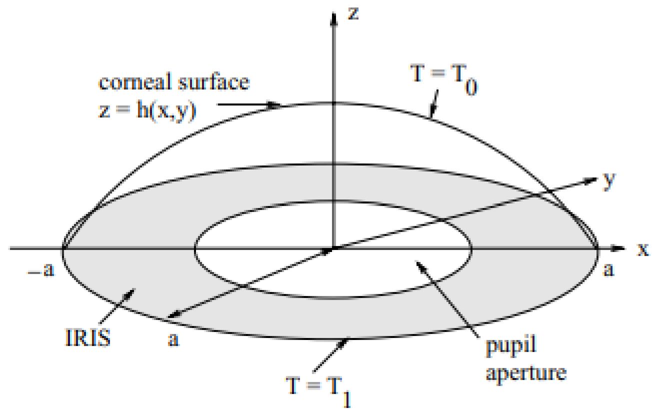


Fig. The geometric representation of the antrerior chamber of the Human Eye[2]

The underlying partial differential equations model for the buoyant flow of Aqueous Humour in the Anterior Chamber has been cited from [1] [Alistair] with further explanation and analysis performed in this paper to understand the relationship between the temperature difference of cornea and iris and the intraocular pressure. The major effort has been put in deducing the differential equations from the very standard Navier - Stokes equations with respect to the geometry of the eye.

We begin by considering the velocity vector of Aqueous Humour fluid in the anterior chamber to be given by:

$$\vec{q} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$$

Let the temperature of Aqueous Humour be T and the pressure of Aqueous Humour be p
and T_0 and T_1 denote the temperatures of the posterior surface of cornea and the anterior surface of iris respectively.

From the Navier-Stokes Equation we have:

$$\rho_0\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right) = \rho_0\vec{g} - \nabla p + \mu.\nabla^2\vec{q}$$

This can be further simplified as:

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \;=\; \vec{g} - \frac{\nabla p}{\rho_0} + \frac{\mu}{\rho_0}\,.\nabla^2\vec{q}$$

Now, as the flow of Aqueous Humour in the Anterior Chamber of the eye is a steady-state flow, we have:

$$\frac{\partial \vec{q}}{\partial t} = 0$$

Also the Aqueous Humour is an Incompressible Fluid; hence the continuity equation ($\nabla.\vec{q} = 0$) holds true.

$\vec{g}$ acts only along the positive X-axis. As there is a temperature difference ($T_1 - T_0$) between the posterior surface of cornea and the anterior surface of iris; a buoyant flow takes place.

We consider the Boussinesq's approximation for buoyancy. Hence, along the X-axis, $\vec{g}$ is replaced by $g(1 - \alpha\,(T - T_0)\,)$

It is to be noted that along Y-axis and Z-axis, $g = 0$ and the term $\frac{\mu}{\rho_0}$ is taken as ν or Kinematic Viscosity.

As the change in velocity only occurs along the Z-axis, $\nabla^2\vec{q}$ is taken as u_{zz}, v_{zz}, w_{zz} along X-axis, Y-axis and Z-axis respectively.

Hence, we get:

1) For flow along X axis,

$$g(1 - \alpha(\,T - T_0)\,) - \frac{p_x}{\rho_0} + \nu u_{zz} = 0 \qquad \text{---------- (1)}$$

2) For flow along Y axis,

$$-\frac{p_y}{\rho_0} + \nu v_{zz} = 0 \qquad \text{------------ (2)}$$

3) For flow along Z axis,

$$-\frac{p_z}{\rho_0} + \nu w_{zz} = 0 \qquad \text{-------------- (3)}$$

Now it is to be noted that the velocity component along Z-axis changes at a constant rate. Hence,

$$w_{zz} = 0 \qquad \text{----------------- (4)}$$

From equations (3) and (4), we get :

$$-\frac{p_z}{\rho_0} = 0 \quad i.e., \quad p_z = 0 \qquad \text{------------------- (5)}$$

Expanding the continuity equation we get :

$$u_x + \mathrm{v}_y + w_z = 0 \qquad \text{--------------------- (6)}$$

To account for the constant change in temperature of Aqueous Humour along the Z- axis , we take $T_{zz} = 0$ ---------------------- (7)

Hence finally, from equations (1), (2), (5), (6) and (7), the final set of Differential Equations becomes:

$$g(1 - \alpha(T - T_0)) - \frac{p_x}{\rho_0} + \nu u_{zz} = 0$$

$$-\frac{p_y}{\rho_0} + \nu \mathrm{v}_{zz} = 0$$

$$p_z = 0$$

$$u_x + \mathrm{v}_y + w_z = 0$$

$$T_{zz} = 0$$

3.5 Boundary Conditions

The above set of Differential Equations is subjected to the following Boundary Conditions, obtainable from the 3D model of the anterior chamber given above:

At $z = 0$

$u = v = w = 0$, $T = T_1$

At $z = h(x, y)$

$u = v = w = 0$, $T = T_0$

Here, $h(x, y) = h_0\sqrt{1 - \frac{x^2}{a^2}}$

4. Results/Key Findings

Under the simplest assumptions made earlier , we find that $v = 0$, so that flow takes place in two-dimensional vertical (x , z) - slices of the anterior chamber. The system of differential equations is solved by applying Computational Fluid Dynamics, a branch of fluid mechanics that uses numerical analysis and data structures to analyze and solve problems that involve fluid flows. This invloves using online CFD softwares, creating a meshwork to describe the geometry and many more steps. Here, we avoid such computations and directly cite [1] [Alistair] to obtain the solution of the system of differential equations. The solution is given as follows:

$$\boldsymbol{p = p_a + g\rho_0 \frac{(x + a)(2 - \alpha(T_1 - T_0))}{2}}$$

$$\boldsymbol{T = T_1 + \frac{z}{h_0}(T_0 - T_1)}$$

Now, the term $T_0 - T_1$ corresponds to the difference between the temperatures of the corneal surface and the iris. This term is approximately equal to 2◦C .

The system of solution equations gives us the pressure and temperature at each point of the X-Z plane.

The pressure (p) can be integrated throughout the domain, i.e., from $x = -a$ to $x = a$ to obtain the net pressure of the anterior chamber of the eye. This net pressure can further be compared with the critical pressure, i.e. 21 mm Hg.

Remark: We don't get into the complications of such calculations here as the purpose of this paper is to discuss how partial differential equations can be used to model complex variables like the intraocular pressure of the eye and not carrying out such complex computations.

5. Conclusion

In this paper, we have discussed how standard Navier - Stokes equations defining fluid dynamics can be mathematically modified as per required to model the intraocular pressure in the human eye.

Open-angle glaucoma is a multifactorial disease, and among the several risk factors, a high intraocular pressure represents the most consistent and the only one that can be modified in order to provide a significant impact over the course of the disease.

High intraocular pressure is significantly associated to the onset and the progression of open angle glaucoma and hence, relatively straightforward mathematical modelling of the intraocular pressure can often serve as a useful clinical indicator for the detection of glaucoma.[6]

6. References

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Annexure

We are just a couple of inquisitive math undergraduate students (not biologists / Ophthalmologists) and the analysis is merely based on our understanding of the subject matter, keeping in mind, numerous assumptions. As the reader might have noticed by now, we have avoided any plausible computations or simulations as we are not quite well equipped with the knowledge / skills to carry out such actions. Moreover, the purpose of this paper was just to portray how differential calculus, with some basic knowledge of fluid dynamics, can predict diseases like open - angle glaucoma.

SRI VENKATESWARA INTERNSHIP PROGRAM FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

Project Report -2020

"The Corona Curve"

Sri Venkateswara College

University of Delhi

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SRIVIPRA PROJECT 2020

Title : The Corona Curve

Name of Mentor: **Dr. Swarn Singh** **Name of Department:** **Mathematics Department** **Designation:** **Associate Professor**	

List of students under the SRIVIPRA Project

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2	Prashasti Sharma	Bsc. (Hons.) Mathematics	
3	Prassanna Nand Jha	Bsc. (Hons.) Mathematics	

4	Rajat Kumar	Bsc. (Hons.) Mathematics	
5	Vanshika Bansal	Bsc. (Hons.) Mathematics	

Signature of Coordinator

SRIVIPRA 2020

Signature of Mentor

Certificate

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project entitled "**The Corona Curve"**. The participants have carried out the research project work under my guidance and supervision from June 15, 2020 to June 30, 2020. The work carried out is original and carried out in an online mode.

Signature of Mentor

(Dr. Swarn Singh)

Acknowledgements

We wish to extend our heartfelt gratitude to Dr. Swarn Singh from Department of Mathematics, Sri Venkateswara College, University of Delhi for his regular guidance, without which we would not have been able to accomplish this project.

15th August 2020

Anant Narayanan, Prassanna Nand Jha

Prashasti Sharma, Rajat Kumar

Vanshika Bansal

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Abstract

The COVID - 19 pandemic was declared as a public health emergency of international concern by the World Health Organisation in the second week of March 2020. Originated from China in December 2019, it has already caused havoc around the world, including India. The first case in India was reported on 30th Jan 2020, with the active cases crossing 1,60,000 on the day this paper was written. For a better understanding of the evolution of COVID - 19 in the country, the SEIRD model has been used in this paper. With the current data available, we predicted that for the Best-Case, Worst-Case and Present Scenario, the Number of infections is likely to peak in Mid-November 2021, Early-August 2020 and Early-March 2021 respectively, after which the number of active infections would go on decreasing till it plummets to 0.

Keywords:

1) Latent period: The time interval between when an individual is infected by a pathogen and when he/she becomes capable of infecting other susceptible individuals. The Latent period for Covid-19 is taken to be 3 days.

2) Mortality rate: The ratio of number of deaths due to Covid-19 to the total number of closed cases.

3) Per-capita transmission rate: It is the rate at which susceptible population is exposed to the disease. It depends on three parameters, the testing rate, number of contacts per person and probability of transmission.

4) Susceptible: Those who can contract the disease.

5) Exposed: Those who are infected but do not spread/transmit the disease.

6) Infectives: Those who are infected and, also spread the disease.

7) Recovered: Those who have recovered from COVID-19.

8) Deceased: Those who died due to COVID-19.

1. Introduction

On 31st December, 2019 The Wuhan Municipal Health Commission made the first public announcement of a pneumonia outbreak of unknown cause , confirming 27 cases enough to trigger an investigation. Today, the COVID - 19 outbreak has spread to more than 200 Countries with more than 8.66 million cases worldwide, as of 20th June, 2020.India reported the first confirmed case of the coronavirus infection on 30 January 2020, in the state of Kerala. On 24 March 2020, The Government of India under Prime Minister Narendra Modi ordered a nationwide lockdown, which continued in the form of multiple phases. The current phase 5, enforces some restrictions only for the containment zones, and is scheduled to end on 30 June, 2020.

Mathematical equations are widely used to model the nature and impact of global pandemics in the society. The SIR model is the classically adopted mathematical model to analyse and predict evolution of a disease. Its one of the variant SEIRD is considered to be the best modelling technique for COVID-19, where exposed population plays an important role. The model also accounts for the deceased population, which is quite high in case of India. Parameters and indicators that quantify the growth and spread of disease in India have subsequently been assumed and determined.

2. Goal of the study

2.1 To formulate a system of Ordinary Differential Equations to model and study the spread of Coronavirus in India.

2.2 To analyse the different scenarios of the Nationwide lockdown implemented (taking into account the limitations) by using Numerical Methods to solve the system of Differential Equations.

3. Methods

3.1 Modelling Approach

We utilized the Susceptible-Exposed-Infective-Recovered-Dead (SEIRD) model to forecast the COVID-19 outbreak in India, based on daily observations. We used the Wolfram Mathematica system to model the spread and performed an in-depth analysis of the different scenarios of the nationwide lockdown. The data is extracted from verified sources such as Worldometers.info, Wikipedia, WHO and covid19india.org, a website authorized by the Indian Government. The site reports confirmed Covid-19 Cases, as well as recovered and deaths for the affected regions.

3.2 Why the SEIRD Model?

The simplest and the most common mathematical model used for the study of spread of diseases such as influenza is the Susceptible - Infected - Recovered model or the SIR model. Besides, deadly diseases like Ebola need yet another compartment, the deceased D, to account for the fatalities, which is significantly high in such cases. But, diseases like COVID - 19 have a significant latent period, during which an individual is 'infected' but not yet an 'infective'. It is important to take the latent period into consideration as it affects the time at which the number of infectives reaches its peak.

Hence, they cannot be perfectly modelled using just these four compartments.

This delay between the acquisition of infection and the infective state can be incorporated within the traditional SIRD model by adding a latent/exposed population, E, which can be seen as a pathway between being susceptible and being an infective. The latent period for COVID-19 is taken to be 3 days, which means that it takes 3 days for an exposed individual to become an infective.

Moreover, it is to be noted that all the exposed individuals eventually become Infectives, but since the latent period for COVID-19 is short, the rate at which the exposed individuals become Infectives is much greater than the rate at which the susceptible population is exposed to the disease. This, in turn, accounts for some extent of asymmetry between the graphs of the Infectives and Exposed populations as the reader might notice further.

The following diagram clearly depicts the SEIRD model, taken into consideration.

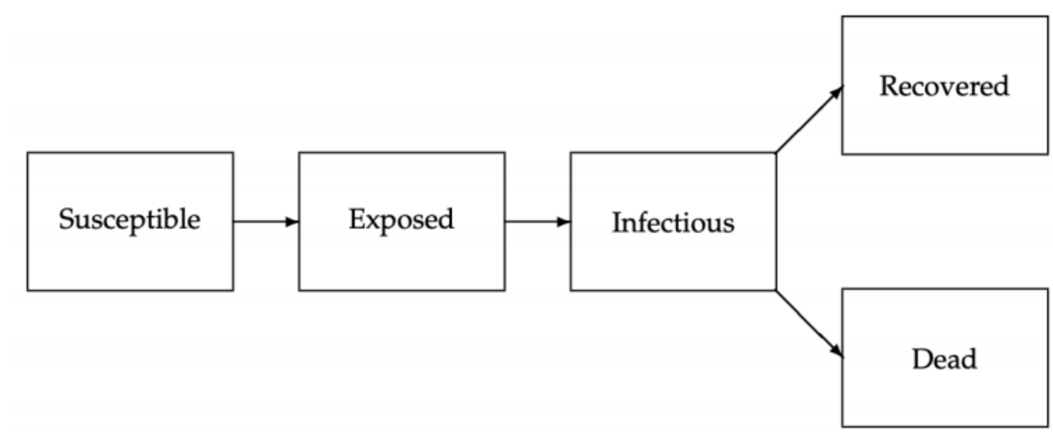

Figure 1. A compartmental diagram of the SEIRD model

3.3 Basic Assumptions

For the purpose of modelling, we made some key assumptions, as follows:

1) The natural birth and natural death rates were ignored.

2) All the elements of the population are identical and have an equal chance of contracting the disease.

3) Migration of people, in and out of the country is not taking place.

4) COVID -19 does not have strong seasonality in its transmission.

5) Once recovered, people become immune to the disease.

6) The average time for recovery is 21 days.

7) The average time taken for a patient to die due to Covid-19 is 25 days.

8) In India, 94 % of the infected people tend to recover, whereas 6% die due to the virus.

9) Once an infective person comes in contact with a susceptible person, the latter is exposed to the disease.

10) The population of India was taken to be constant at 1.3 billion, but was scaled down to the unit value 1, to make the analysis easier.

11) The values of all the other parameters were scaled down accordingly.

12) The trajectory of the infection count has been modelled, starting from 10 Jan,2020.

3.4 Parameters Used

Let,

'a' be the per capita transmission rate at which susceptible is exposed to disease.

$a = t \times n \times p \quad, p = 1$

where

t is the testing rate for Covid-19

n is the numbers of contacts per person

p is the transmission probability

'b' be the per capita rate at which exposed population becomes infectives.

$b = \frac{1}{Latent\ Period} = \frac{1}{3} = 0.333$

'c' be the per capita rate at which infectives recover from COVID-19

$c = 0.94 \times (\frac{1}{Time\ Period\ of\ Recovery}) = 0.94 \times (\frac{1}{21}) = 0.04476$

'd' be the per capita rate at which infectives die due to COVID-19

$d = 0.06 \times (\frac{1}{Time\ Taken\ for\ Death}) = 0.06 \times (\frac{1}{25}) = 0.0024$

3.5 The Differential Equations

$$\frac{dS}{dt} = -a \times S \times I$$

$$\frac{dE}{dt} = a \times S \times I - b \times E$$

$$\frac{dI}{dt} = b \times E - c \times I - d \times I$$

$$\frac{dR}{dt} = c \times I$$

$$\frac{dD}{dt} = d \times I$$

3.6 Initial Conditions

We assume that in the beginning, majority of India's population was susceptible with 90 % of the remaining being exposed and only 10 % being infectives. Besides, there were no recovered or deceased individuals. With initial values scaled down on a scale of 0 to 1, the initial conditions for all the five compartments are then given by:

S(0) = 0.999999

E(0) = 0.0000009

I(0) = 0.0000001

R(0) = 0

D(0) = 0

4. Results

4.1 The Three Scenarios

Now, we will model the 3 different scenarios of the nationwide lockdown implemented in our country, with the transmission rate 'a' varying for each scenario.

In these plots, the X- axis represents the number of days since 10th Jan, 2020 and the Y- axis denotes the population squished on a scale of 0 to 1.

4.1.1 Present Scenario

Here we model the current scenario of the lockdown. Currently, the average testing rate is 0.5 % and the average number of contacts per person is taken to be 17 due to the sudden implementation of lockdown and some people not abiding by the rules. Also, some people are not following the 'quarantine' guidelines which increases the number of contacts per person.

The transmission probability is 1.

Hence, $a = 0.005 \times 17 \times 1 = 0.085$.

We model this scenario using the system of differential equations formed above in the Wolfram Mathematica System and solve them using Numerical Methods.

The plots obtained are shown below**:**

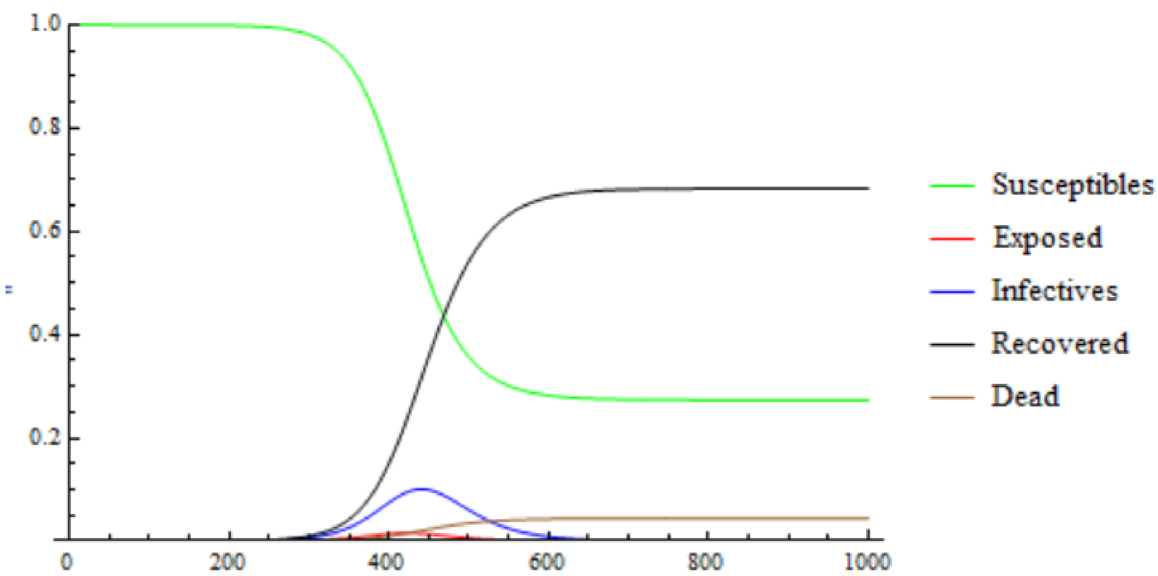

Figure 2. The time series plot for Present Scenario

The graph shows the trend followed by all the five compartments as time proceeds. However, the present state is not observable now, so we need to zoom in near the origin.

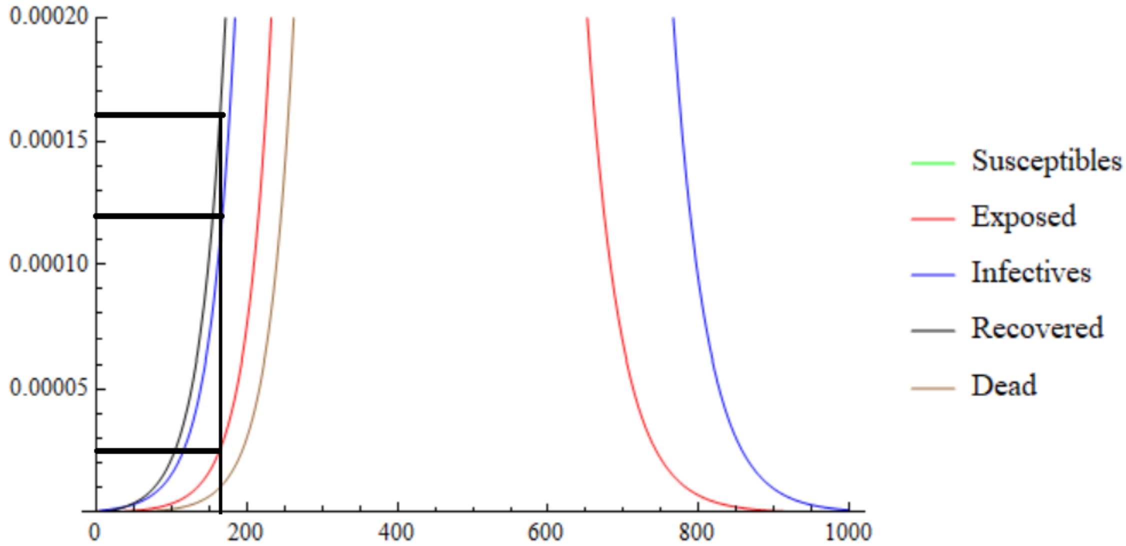

Figure 3. Figure 2 zoomed in

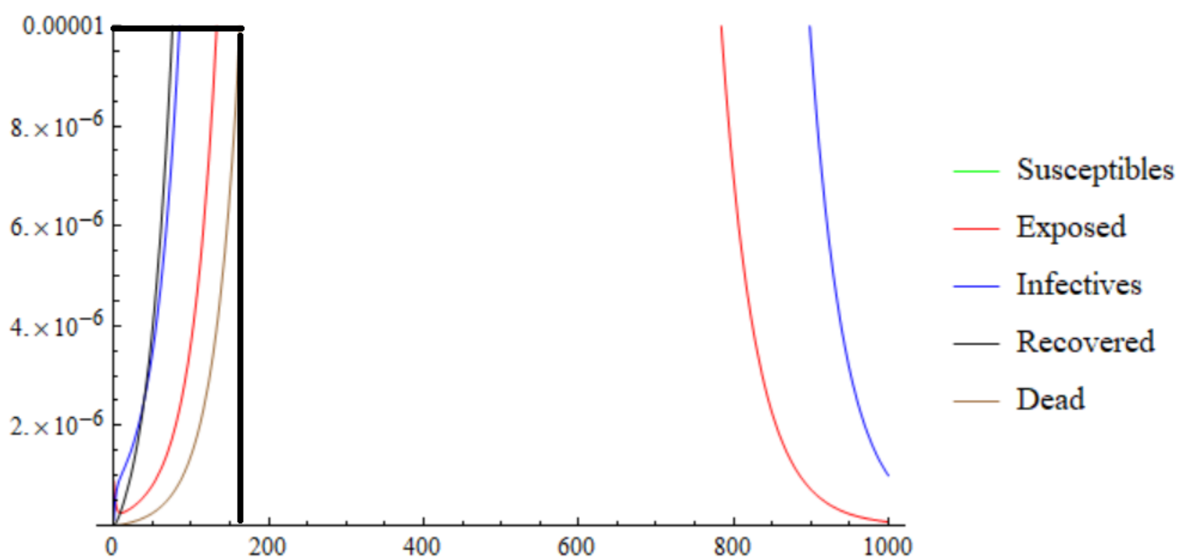

Figure 4. Figure 2 zoomed in further

These images show that almost 160 days into the timeline, i.e. on about 20th June, 2020:

1) The number of infectives = 0.00016 which is equivalent to 1,63,651

2) The number of exposed = 0.000025 which is equivalent to 32500

3) The number of recovered patients = 0.00012 which is equivalent to 2,05,183

4) The number of deaths = 0.00001, which is equivalent to 12,605

Interestingly, these figures exactly match with those provided by the government websites.

To further confirm with our model, we also checked the figures at almost 100 days into the timeline, i.e. infectives being 0.00001, equivalent to 12, 605. Even here, the government figures seemed to match with our calculations.

Also, according to this model, the number of infectives is likely to peak around Early-March, 2021 and it is predicted that about 18 % of India's population will become infectives at that point in time.

Here again, by 'infectives', we are referring to the people capable of infecting others.

4.1.2 Best Scenario

Here we model the best-case scenario for the nationwide lockdown, taking into account the limitation of healthcare resources. India's testing rate is pretty low as compared to some of the other countries. So, this scenario would involve a substantial increase in the testing rate, say it doubles up to 1 %.

People would follow the lockdown rules diligently and exposed individuals will quarantine themselves lowering the number of contacts per person to, say 7.

The transmission probability would remain 1.

Hence, $a = 0.01 \times 7 \times 1 = 0.07$

We model this scenario using the system of differential equations formed above in the Wolfram Mathematica System and obtain the corresponding plot:

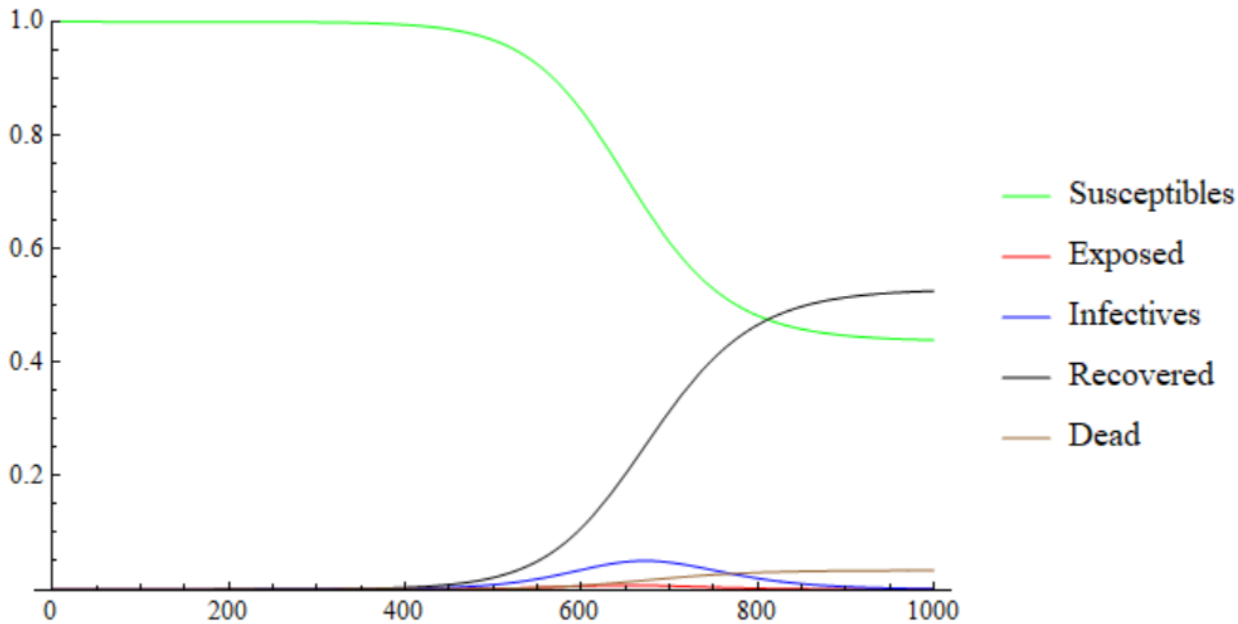

Figure 5. The Time Series Plot for Best Case Scenario

Here, the Best-Case Scenario involves the number of active infections to peak in Mid-November, 2021. Also, note that the 'infective' curve is flatter as compared to the previous case. The peak would take longer to occur, but the number of infections would be manageable for the healthcare system, not overburdening it.

The fraction of population becoming infectives in this case, would be least amongst all the 3 scenarios. It is expected that only 7 % of India's population will catch infections when at peak. Hence this is the most favourable scenario for the COVID-19 spread in India.

4.1.3 Worst Scenario

Here we model the worst-case scenario for the nationwide lockdown. In this case, the testing rate would be very low, say 0.3 %. The lockdown rules would not be strict and people would be able to leave their homes for work.

Contact tracing would not take place and hence the exposed individuals are not quarantined. Thus the number of contacts per person would increase to , say 50. The transmission probability would remain 1.

Hence, $a = 0.003 \times 50 \times 1 = 0.15$.

We model the system of differential equations using the Wolfram Mathematica System and obtain the corresponding plot:

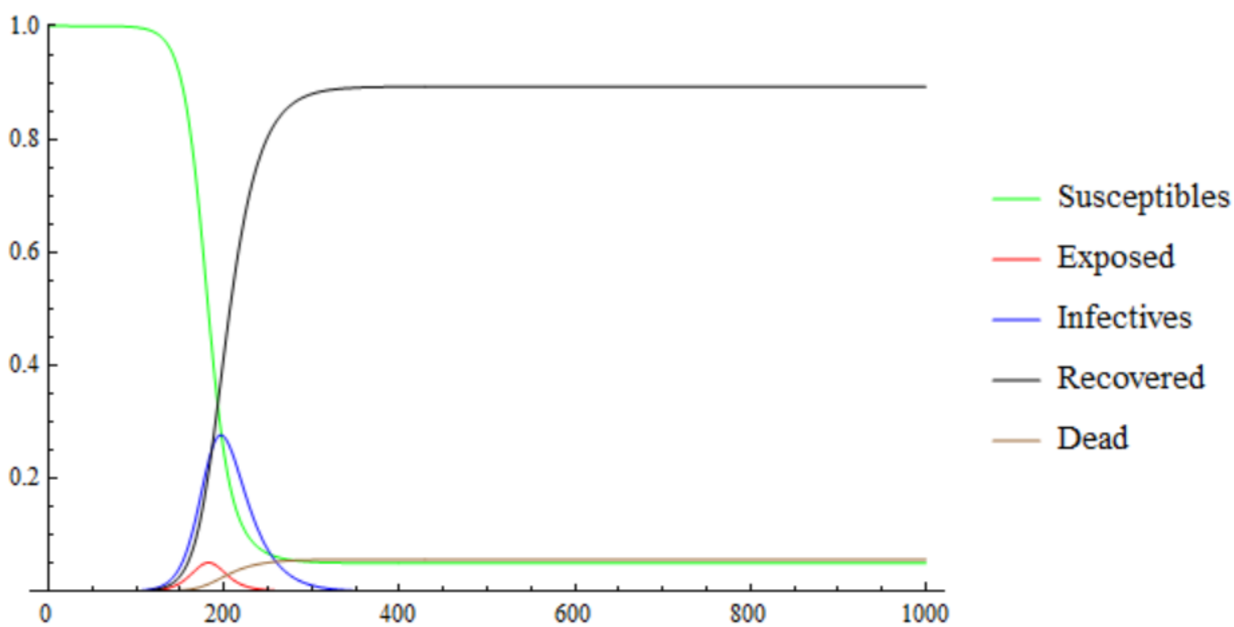

Figure 6. The Time Series Plot of Worst-Case Scenario

According to the Output obtained in Mathematica, The Worst-Case Scenario involves the peak of infections to happen in Early August, 2020. Also the 'Infective' Curve is steeper as compared to the previous 2 cases, meaning that the peak number of infective people would be higher as compared to the previous 2 models.

It may appear to the reader that the peak occurs faster in this case, so it should be favourable. But in this case, there is a sudden growth in cases and a greater fraction of individuals are infected, causing chaos. The Healthcare infrastructure is not well equipped to deal with this sudden surge in the number of cases, and hence, it would lead to greater number of deaths and chaos. In this case, as much as 25 % of India's population become infectives when their number is at peak. For this reason, the scenario is the least favourable.

5. Conclusion

With the current data available, we predict that

1) For the present scenario, the number of infections is likely to peak in Early March 2021.

2) For the best- case scenario, the number of infections is likely to peak in Mid November 2021.

3) For the worst- case scenario, the number of infections is likely to peak in Early August 2020

after which the number of active infections would go on decreasing till it plummets to 0.

Also, to limit the number of infections, people need to follow the social distancing norms throughout. Washing their hands regularly, wearing masks, are some of the norms people need to follow.

6. References

1) www.worldometers.info/coronavirus/ [Last accessed: 19th June, 2020]

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5) Barnes, Belinda Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor Francis Group.

6) https://arxiv.org/abs/2005.12777 : SEIRD Model for Qatar Covid-19 Outbreak: A Case Study written by Ryad Ghanam, Edward L. Boone, Abdel-Salam G. Abdel-Salam

Annexure

We are just a bunch of inquisitive undergraduate students (not epidemiologists) and the analysis is based on elementary mathematical modelling techniques, keeping in mind, numerous assumptions. COVID-19 is still an infectious disease with unclear or unknown properties, so an accurate result can only be obtained once the outbreak is successfully contained in the country. Hence, these dates and numbers must not be taken for any administrative decisions.

Abbreviations

COVID-19: Coronavirus disease 2019;

SEIRD: Susceptible - Exposed - Infected - Recovered - Dead

SRI VENKATESWARA INTERNSHIP PROGRAM
FOR RESEARCH IN ACADEMICS
(SRI-VIPRA)

Project Report -2020

"Partial Differential Equations in oil reservoirs"

Sri Venkateswara College
University of Delhi
Dhaula Kuan
New Delhi -110021

SRIVIPRA PROJECT 2020

Title : Partial Differential Equations in oil reservoirs

Name of Mentor : Dr. Swarn Singh **Name of Department : Department of Mathematics** **Designation : Associate Professor**	

List of students under the SRIVIPRA Project

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2	Dolly Goyal	B.Sc (Hons.) Mathematics Semester II	
3	Shriya Koul	B.Sc (Hons.) Mathematics Semester II	

Signature of Coordinator **Signature of Mentor**

Certificate

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project entitled "Partial Differential Equations in oil reservoirs". The participants have carried out the research project work under my guidance and supervision from July1, 2020 to July 31, 2020. The work carried out is original and carried out in an online mode.

Signature of Mentor

Acknowledgement

We acknowledge Dr. Swarn Singh, University of Delhi, New Delhi for helpful discussion regarding technicalities of the method used in this model.

15th August, 2020 Prashasti Sharma, Dolly Goyal, Shriya Koul

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Abstract

Importance of oil yield is no new to this fuel driven world. Reservoir modeling is the analytical key to identify the various dynamic processes occurring in the reservoir .
The reservoir model is considered dynamic when the fluid movement within the system is taken under consideration by including rock – fluid properties such as relative permeabilities, fluid saturations, connate water saturations and aquifer properties . Once the reservoir is modeled in every such detail, calculating the movement of the reservoir fluids (oil, gas, etc.,) under available driving force which becomes feasible. The method is known as flow simulation.

Reservoir flow simulation is a very prolific task to predict the performance of reservoir under study. The reservoir simulation consists of set of non – linear partial differential equation with appropriate initial and boundary conditions that describe the dynamical fluid flow behaviors within the reservoir over the time.

1. Key Words

1. Ψ= Porosity
2. S = Saturation
3. P_c = Capillary Pressure
4. K_r = Relative permeability
5. λ = Mobility
6. C_f = Compressibility
7. V = Volume
8. P = Pressure
9. K = Permeability of a medium
10. q = Instantaneous flow rate
11. Q = Total discharge / Flux
12. μ. = Dynamic Viscosity of a fluid
13. ∇P = Pressure gradient

14. K' = Intrinsic Permeability tensor
15. g' = gravity vector
16. ρ = fluid density
17. C_t = Total Compressibility
18. Z = vector function of (x, y, z) pointing in the direction of gravity

Subscripts used: - w, o = water and oil respectively

2. Introduction/ Background

2.1 Some basic / important physical properties and their definitions:-

1. **Porosity (Ψ):** Rock porosity represents the void space in the porous media, where the fluids get accumulated. Porosity is defined as the ratio of pore volume to the total bulk volume of the rock.

$$\boldsymbol{\Psi} = \frac{\boldsymbol{Pore\ Volume}}{\boldsymbol{Total\ Volume}}$$

Porosity is dependent on the fluid pressure if the rock is compressible. There are, primarily, two types of porosity, total and effective porosity. The total porosity represents the ratio of total volume of the pore space to the bulk volume, whereas, the effective porosity is the ratio of interconnected pore volume to the bulk volume.

2. **Permeability (K):** The ability of a rock to transmit fluid through interconnected pore space is termed as permeability. When the reservoir rock is 100 % saturated with a single phase fluid it is termed as absolute permeability. Effective permeability is the ability of the rock to transmit fluid in presence of other immiscible fluids. Permeability is also a rock property and therefore, varies at different locations and even at the same location with the flow directions. It is strongly correlated to porosity since the interconnections and orientations of pores are vital to fluid flow.

3. **Fluid Saturation (S):** Saturation is expressed as that fraction, or percent, of the pore volume occupied by a particular fluid phase (oil, gas, or water) in the void space. Saturation is mathematically defined as:

$$S = \frac{\textit{Total volume of the fluid}}{\textit{Pore volume}}$$

All saturation values are based on pore volume and not on the gross reservoir volume. The saturation of each individual phase ranges between 0 to 100%. For a two phase fluid flow of oil and water, the sum of the saturations is 100%, i.e.

$$S_o + S_w = 1$$

where, S_o, and S_w corresponds to fractional saturation of oil and water respectively.

4. **Capillary Pressure (P_c):** A discontinuity in pressure exists between the two fluids when two immiscible fluids are in contact, which depends upon the curvature of the interface separating the fluids. This pressure difference is referred as the capillary pressure and mathematically defined as Pc:

$$\mathbf{Pc(Sw)} = \boldsymbol{P_o - P_w}$$

The pressure excess in the non-wetting fluid is the capillary pressure, and is a function of saturation.

5. **Relative Permeability (k_r):** When two or more fluids flow at the same time, at a specific saturation, the ratio of the effective permeability of the corresponding phase to the absolute permeability is termed as relative permeability of the corresponding phase. The relative permeability is affected by the pore geometry, wettability, fluid viscosity and saturation history. Relative permeability is dimensionless and varies between zero and one.

$$\boldsymbol{K_r = \frac{Effective\ Permeability}{Absolute\ Permeability}}$$

When the reservoir displacement process is dominated by gravity, the relative permeabilities are functions of saturations, and it's only essential to know the endpoint saturations, the irreducible water saturations and residual oil saturations. The residual oil saturation is an important parameter used to determine the overall oil recovery.

6. **Mobility (λ) :** The mobility of fluid phase is defined as the ratio of the effective phase relative permeability to the phase viscosity. The mobility is expressed as

$$\boldsymbol{\lambda = \frac{Effective\ phase\ permeability}{Phase\ viscosity}}$$

7. **Phase :** Phase is a homogeneous region of a fluid separated from another phase by an interface, e.g., oil, gas or water. Two phases are said to be immiscible if both the phases cannot be mixed in any proportion to form a homogeneous solution.

8. **Compressibility (C_f) :** The change in volume (V) or density (ρ) of the fluid with respect to the pressure (P) is termed as the compressibility of the fluid and is expressed as

$$C_f = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T$$

2.2 Assumptions :-

1. Parameters such as reservoir boundary and sectoring are neglected.
2. Physical rock fluid properties such as permeability, saturation, etc., are included to understand the fluid flow system and that they depend on pressure.
3. Composition and Permeabilities of fluids remain constant over time.
4. Three dimensional flow of liquid has been considered.
5. Non – Darcy flow of liquid has been neglected.
6. Parameters like relative permeability and capillary pressure depends on saturation only (two – phase Darcy approach).
7. Viscous coupling between fluid phases is ignored.
8. Coupling Pressure is assumed to be equal to the difference in wetting and non – wetting pressures.

2.3 Darcy's Law :-

Darcy Law is an equation that describes the flow of a liquid through a porous medium.

The Law provides the simple proportionality relationship between the instantaneous flow rate q through a porous medium, the permeability K of a medium, the dynamic viscosity of the fluid and the pressure drop over the given distance in the form :

$$\mathbf{q} = -\frac{K}{\mu}\,\Delta P$$

The above equation is the governing equation for single – phase fluid flow in a phase medium.

The integral for the same is given by :-

$$\mathbf{Q} = \frac{KA}{\mu L}\,\Delta P$$

Where Q is the total discharge / flux (unit volume per time)

For 3 – D Single – Phase flow :-

We have,

$$\mathbf{v} = \mathbf{q} = \frac{Q}{A} = -\frac{K'}{\mu}\,(\nabla P - \rho g')$$

where, ∇P = *Pressure Gradient*
K' = **Intrinsic Permeability tensor** assumed to be a property of porous solid.
And g' = gravity vector

However, for Multi – phase flow, the equations become complicated. The coefficients become phase specific and permeability depends on saturation and is derived in the form given below:-

$$\mathbf{v}_\alpha = \frac{K'_\alpha}{u_\alpha}(\nabla P_\alpha - \rho_\alpha g'), \quad \mathbf{x} \in \boldsymbol{\Omega}$$

(α = w for wetting phase and α = n for non – wetting phase).

$$K'_{\alpha} = K'_{rd} \, . \, K'$$

2.4 Single Phase flow and Two Phase flow:-

Fluid motions in Porous media are governed by the fundamental laws that are based on conservation of mass, momentum and energy.

The flow inside the reservoir is assumed to be single phase when we want to ignore the complexities, in equations created by including more physical states of fluids, so we continue only consider single – state (oil) fluid flowing through the reservoir. The equations depicting single – phase flow , stands insufficient to model the simultaneous interactive flow of two or more phases (gas, water, oil, etc.,). So, we need to derive multi – phase flow equation in porous media.

2.5 Mass Conservation:-

The Conservation of mass principle implies that the rate of accumulation of mass inside a control volume and the net rate of the outflow of amount of mass across the control surface equal to zero. The total amount of matter in a given process is fixed, but it may change from one form to another.

Mathematically,
$$\frac{\partial \rho_v}{\partial x} = -\Psi \frac{\partial \rho}{\partial x} + q$$

$$\Psi \frac{\partial \rho}{\partial x} = -\frac{\partial \rho_v}{\partial x} + q$$

Using Divergence theorem and then integrating spatially, we obtain the partial differential form for single – phase flow:-

$$\frac{\partial (\emptyset \rho)}{\partial t} = -\nabla . (\rho v) + \mathbf{q}$$

Combining Darcy’s Law equation for empirical fluid velocity,

$$\partial \frac{\Psi \rho}{\rho t} = \nabla . \frac{\rho K}{\mu} (\nabla P - \rho g \nabla Z) + q$$

Now, using

$$c = \frac{-1}{\rho}\frac{\partial \rho}{\partial P}\bigg| T$$

and solving the equation , we get

$$P = P_0 + \frac{1}{c}\ln\left(\frac{\rho}{\rho_0}\right)$$

where ρ_0 is the density at Pressure P_0 .

Using, Chain rule in A, we get the following parabolic Partial Differential Equation

$$\partial \frac{\Psi\rho}{\rho t} = \nabla . \frac{K}{\mu c} (\nabla P - \rho^2 cg \nabla Z) + q$$

Now applying the required boundary condition over it we get,

$$\rho \, \mathrm{v} . v /_{\partial\Omega} = g_1 (x , t) \qquad , \mathrm{x} \in \partial\Omega \quad , \mathrm{t} \in [\mathrm{t_o}, \mathrm{t_1}]$$

3. Goal of the study

3.1 Formulating the various factors evolved in fluid flow in the reservoir.

3.2 Identifying the final equation in terms of saturation

4. Method

4.1 Our Approach

We consider two phases: Oil and water flowing simultaneously and different from each other). Water naturally tends to occupy more surfaces, so it mostly wets the porous medium, whereas oil is a non-wetting fluid that does not like to occupy more surfaces.

The expression for the saturation of both fluids (phases) is:

$$S_w + S_o = 1$$

The two phases have different pressures of their own, termed as capillary pressures Pc:

$$Pc(Sw) = P_o - P_w$$

The difference in phase pressures is a function of the wetting- phase separation in order to close the system. (Pressure difference is related to saturation).

As water and oil differ in their densities, viscosity, they have different formulation of Darcy's Law; where we extract volumetric velocity $V_\alpha , \alpha =$ **(w,o)** for both phases ,

$$V_w = -\frac{K_{rw}K}{\mu_w}(\nabla P_w - \rho_w g \nabla Z)$$
$$V_0 = -\frac{K_{r0}K}{\mu_o}(\nabla P_0 - \rho_0 g \nabla Z)$$

The mass balance equations for both of the fluid phases are written as:

For water:-

$$\Psi \frac{\partial(\rho_w S_w)}{\partial t} = \nabla . \frac{\rho_{wKK_{rw}}}{\mu_w}(\nabla P_w - \rho_w g \nabla Z) + q_w - \text{ Equation A}$$

For Oil:-

$$\Psi \frac{\partial(\rho_0 S_0)}{\partial t} = \nabla . \frac{\rho_{0KK_{r0}}}{\mu_0}(\nabla P_0 - \rho_0 g \nabla Z) + q_0 - \text{ Equation B}$$

$$, x \in \Omega \quad , t \in [t_o, t_1]$$

where Ω is our spatial domain and $[t_o, t_1]$ is the time interval taken into consideration.

Pressure and densities for each phase are related by the equation form:

$$C_w = \frac{1}{\rho_w}\frac{\partial_{\rho_w}}{\partial P_w}\bigg|\ T$$

$$C_0 = \frac{1}{\rho_0}\frac{\partial_{\rho_0}}{\partial P_0}\bigg|\ T$$

The equation A and B seem to be a system of parabolic partial differential equations, but they actually are not. This becomes clear when we define the formula of total fluid velocity assuming that phase density, porosity are constant and gravity is neglected.

$$\mathbf{V} = -\mathbf{k}\,\lambda\,(\mathbf{Sw})\ \nabla\rho, \quad \mathbf{x} \in \Omega \quad , \mathbf{t} \in [\mathbf{t_o}, \mathbf{t_1}]$$

where, $\lambda(\mathrm{Sw}) = \frac{K_{r0}\,(Sw)}{\mu_0} + \frac{K_{rw}(Sw)}{\mu_w}$

and, $\mu_w = Total\ mobilty$

Phase mobility for water:-

$$\lambda_w(\mathrm{Sw}) = \frac{K_{rw}(Sw)}{\lambda(\mathrm{Sw})\mu_w}$$

Phase mobility for oil:-

$$\lambda_0(\mathrm{Sw}) = \frac{K_{r0}(Sw)}{\lambda(\mathrm{Sw})\mu_0}$$

Now, using equations A and B and the expression for phase mobilities and fluid velocity, we derive the following :

$$\nabla.\mathbf{V} = -\nabla\,.\,(K\,\lambda(\mathrm{Sw})\ \nabla\ P = \frac{q_0}{\rho_0} + \frac{q_w}{\rho_w}$$

$$\Psi\frac{\partial S_w}{\partial t} - \nabla\,(K\lambda.\lambda_0\lambda_w\frac{\partial\rho_c}{\partial Sw}\nabla\,Sw)\lambda_w \mathrm{V}.\nabla\,Sw = f(q_w\,, P_w\,, \lambda_w\,, q_0\,, P_0\,, \lambda_0)$$

$$, \mathbf{x} \in \Omega \quad , \mathbf{t} \in [\mathbf{t_o}, \mathbf{t_1}]$$

where f represents a simple linear function of the flow properties.

For a compressible fluid, we have,

$$\nabla \mathbf{K}\ \boldsymbol{\lambda}\ \nabla \mathbf{P} = \frac{q_0}{\rho_0} + \frac{q_w}{\rho_w} + \Psi C_t \frac{\partial \rho}{\partial t}$$

where C_t is given by

$$C_t = \frac{1}{\Psi}\frac{\partial \Psi}{\partial \rho} + (S_0 C_0 + S_w C_w)$$

$$, x \in \Omega \quad , t \in [t_o, t_1]$$

5. Conclusion:-

In this report, we have presented fluid-flow movements within the reservoir, by making our emphasis on the various fluid properties and thus, formulating them into Partial Differential Equations.

Both phases, single and two-phase flow have been considered as any of them can occur within the reservoir, and for that, Darcy's Law for both the phases have been extended to explain the fluid dynamics and yield our final resulting equations.

5.1 Limitations:-

1. Geological parameters that can actually affect the reservoir modeling have not been considered in the model to make the model simple. A complex model can be generated involving the geological properties of a reservoir to study in depth the productivity of reservoir.

2. The equation needs to be modified differently for various situational circumstances happening in or around the reservoir. (Example- water flooding during rains)

3. The model can be improved by implying a solution procedure to the partial differential equation used. Example- Gridding system can be used.

4. Non-Darcy flow effects can be added.

5. Various experiments conducted on reservoir fluid-Flow system have stated that parameter set of the two-phase Darcy flow description is incomplete, which can be completed by introducing a new parameter 'the specific fluid-fluid interficial area.

6. While we assumed that Capillary Pressure equals the difference between non-wetting and wetting-phase pressures, some cases report that difference (P_o - P_w) differ by a term that depends on the time rate change of saturation, $S_w \partial t$.

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SRI VENKATESWARA INTERNSHIP PROGRAM
FOR RESEARCH IN ACADEMICS
(SRI-VIPRA)

Project Report -2020

"Partial Differential Equations in Image Processing"

Sri Venkateswara College

University of Delhi

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New Delhi -110021

SRIVIPRA PROJECT 2020

Title: Partial Differential Equations in Image Processing

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Name of Mentor: Dr . Swarn Singh

Name of Department: Department of Mathematics

Designation: Associate Professor

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2	Rajat Kumar	B.Sc.(Hons.) Mathematics	
3	Vanshika Bansal	B.Sc.(Hons.) Mathematics	

Signature of Coordinator **Signature of Mentor**

SRIVIPRA 2020

Certificate

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project entitled "**PDE in Image Processing"**.The participants have carried out the research project work under my guidance and supervision from July 1, 2020 to July 31 ,2020. The work carried out is original and carried out in an online mode.

Signature of Mentor

Acknowledgement

We would like to express our special thanks of gratitude to our team mentor Dr.Swarn Singh for the able guidance and support in completing our project and report making. I would like to extend my gratitude to the Principal of Sri Venkateswara College Dr.S Venkata Kumar for providing us Sri VIPRA internship that was required for the welfare of students.

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Abstract

In recent years, the partial differential equations, both fractional and integer orders, have been recognized as a powerful modeling methodology. They are inspired by problems which arise in diverse fields such as biology, fluid dynamics, physics, differential geometry, control theory, materials science, and engineering.

The purpose of this report is to review some recent developments in methods and application of partial differential equations in image processing . We discuss how nonlinear partial differential equations (PDEs) can be used to automatically produce an image of much higher quality, enhance its sharpness, filter out the noise, etc .

1. Goal of the study

1.1 The research in this dissertation has focused upon image segmentation and its related areas, using the techniques of partial differential equations, variational methods, mathematical morphological methods and probabilistic methods.

1.2 PDEs are discretized by variational techniques, namely by the semi-implicit finite element, finite volume and complementary volume methods in order to get fast and stable solutions.

2. Introduction to PDE

In Mathematics , a partial differential equation is one of the types of differential equations , in which the equation contains unknown multi variables with their partial derivatives .It is a special case 25 of an ordinary differential equation.

A Partial Differential Equation commonly denoted as PDE is a differential equation containing partial derivatives of the dependent variable (one or more) with more than one independent variable. A solution to a partial differential equation is a function that solves the equation or, in other words , turns it into an identity when substituted into the equation. A solution is called general if it contains all particular solutions of the equation concerned.

A PDE for a function u(x1,. . . . . . xn) is an equation of the form.

$$f\left(x_1, \dots, x_n; u, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_n}; \frac{\partial^2 u}{\partial x_1 \partial x_1}, \dots, \frac{\partial^2 u}{\partial x_1 \partial x_n}; \dots\right) = 0$$

The PDE is said to be linear if f is a linear function of u and its derivatives. The simple PDE is given by;

∂u/∂x (x,y) = 0

The above relation implies that the function u(x,y) is independent of x which is the

reduced form of partial differential equation formula stated above. The order of PDE is the order of the highest derivative term of the equation.

In PDEs, we denote the partial derivatives using subscripts, such as

$$u_x = \frac{\partial u}{\partial x}$$

$$u_{xx} = \frac{\partial^2 u}{\partial x^2}$$

$$u_{xy} = \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$$

3. Introduction to Image Processing

Image processing is a method to perform some operations on an image , in order to get an enhanced image or to extract some useful information from it . It is a type of signal processing in which input is an image and output may be image or characteristics/features associated with that image . Nowadays , image processing is among rapidly growing technologies . It forms core research area within engineering and computer science disciplines too . Image processing applied to medical research has made many clinical diagnosis protocols and treatment plans more efficient and accurate. For example, a sophisticated nodule detection algorithm applied to digital mammogram images can aid in the early detection of breast cancer .

3.1 Edge Detection

Edges characterize boundaries and are therefore a problem of fundamental importance in image processing. Edges in images are areas with strong intensity contrasts . A fundamental task in application of computer vision is the edge detection since from the early 1970s . Edge detection refers to the process of identifying and locating sharp discontinuities in an image . Edge detecting an image significantly reduces the amount of data and filters out useless information , while preserving the important structural properties in an image .

Many methods are available for detection of edges . Edge detection operators are grouped into two categories:- Gradient based and Laplacian based . Gradient operators use first order derivatives of the image and laplacian operators uses second order derivatives of the image to find edges.

3.1.1.First order derivative of edge detection is computed as follows

$$\nabla f = \begin{bmatrix} Gx \\ Gy \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Magnitude of the vector is the important quantity in edge detection , denoted with ∇f , where

$$\boldsymbol{\nabla f} = |\boldsymbol{\nabla f}| = (\boldsymbol{G_x}^{\boldsymbol{2}} + \boldsymbol{G_y}^{\boldsymbol{2}})^\wedge \frac{\boldsymbol{1}}{\boldsymbol{2}}$$

Direction of gradient vector is another important quantity of edge . That is ,

$$\boldsymbol{angle\ of\ \nabla f = tan^{-1}}(\frac{\boldsymbol{G_x}}{\boldsymbol{G_y}})$$

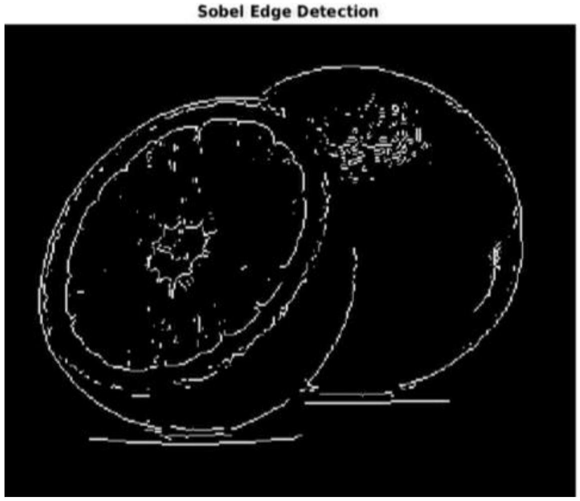
at every pixel location partial derivative of $\partial f/\partial x$ and $\partial f/\partial y$ are computed for calculation of gradient of an image .

This gradient based method includes 3 edge detecting operators:- **Sobel , Prewitt and Roberts .**

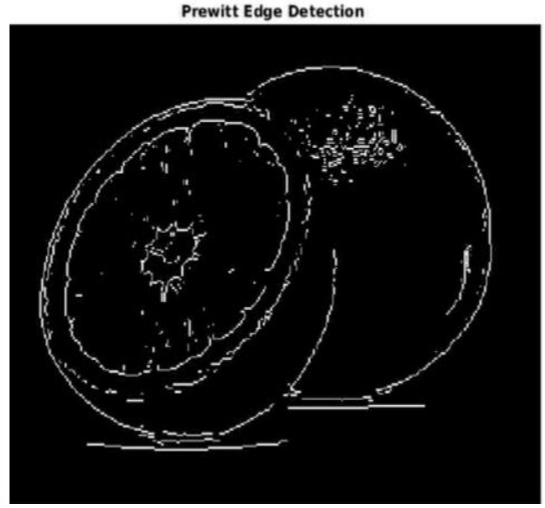
Below are the MATLAB generated outputs of Sobel , Prewitt and Roberts method

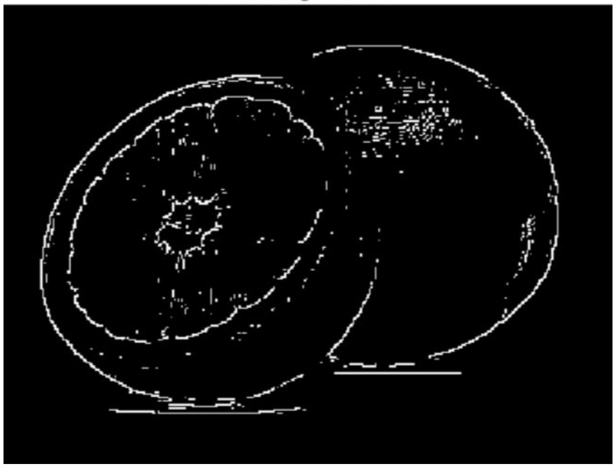
Original image

Sobel Edge Detection



Prewitt Edge Detection





3.1.2 Second order derivative edge detection

The second order derivative , the Laplacian of a 2-D image function f(x,y) is computed as follows:-

$$\nabla^2 = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

The 2-D Gaussian function:- $h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$

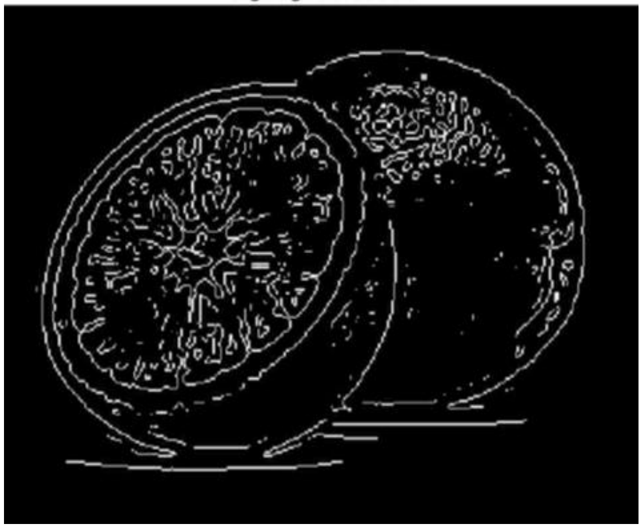
Where σ is the standard deviation , which determines the degree of blurring in the image . The laplacian of h is defined as

$$\nabla h^2(x, y) = -[\frac{x^2+y^2-2\sigma^2}{\sigma^4}]e^{-\frac{r^2}{2\sigma^2}}$$

This laplacian based method include 1 edge detecting operator:- **Laplacian of Gaussian(log)** .

Below is the MATLAB generated output of Laplacian of Gaussian method:-

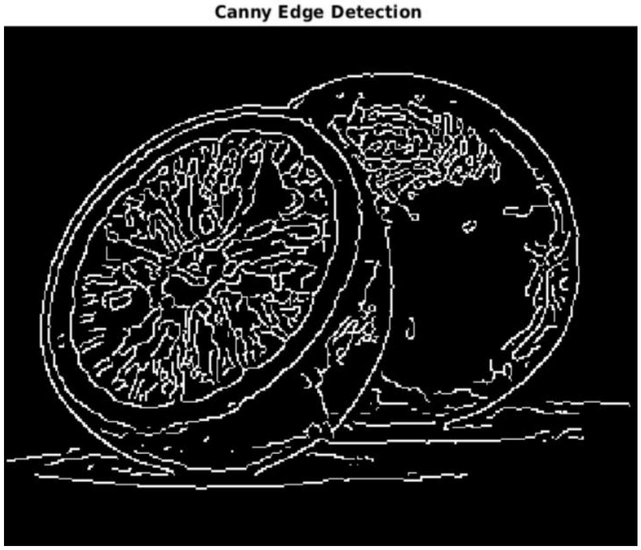
log Edge Detection



We have one more operator for edge detection which is known as **Canny edge operator**.

Here is the MATLAB generated output of Canny edge operator

Canny Edge Detection



3.2 Noise in an Image

While talking about digital image processing , there comes an integrated part of it which is noise . Noise is always present in digital images during image acquisition , coding , transmission and processing steps . Image noise is random variation of brightness or color information in the images captured . It is degradation in image signal caused by external sources . Images containing multiplicative noise have the characteristic that the brighter the area the noisier it is . But mostly it is additive . We can model a noisy image as**,** where

A(x,y)=function of noisy image

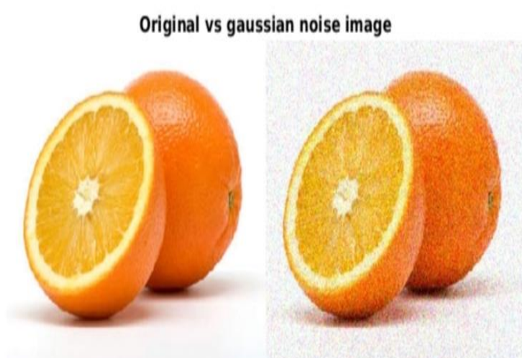
H(x,y)=function of image noise

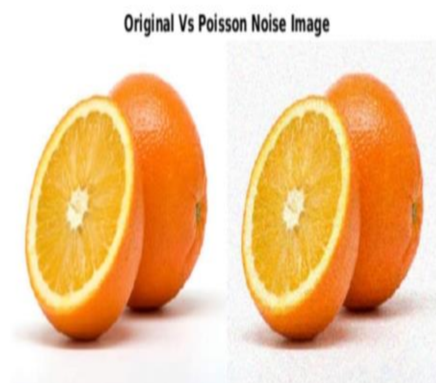
B(x,y)=function of original image

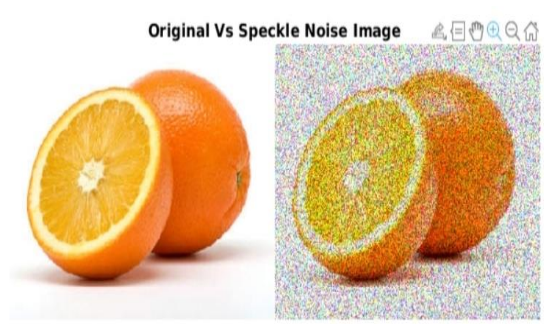
There are mainly 4 types of image noise:- **Gaussian noise , Poisson noise , Salt & Pepper noise and Speckle noise .**

We have used inbuilt functions of MATLAB to add noise in the original image .

Below are the MATLAB generated outputs of all the four noise in an image





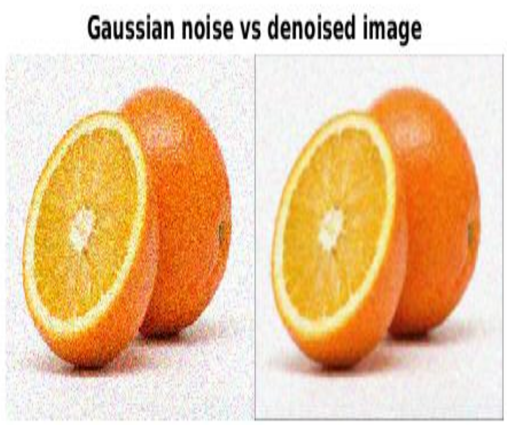


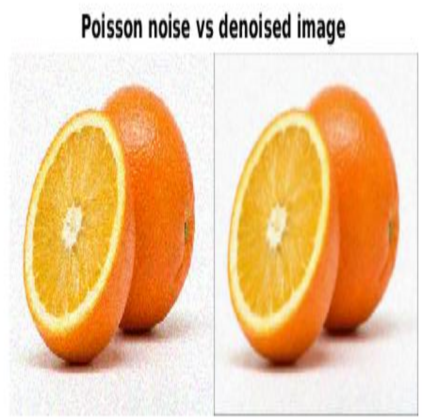
3.3 Denoising

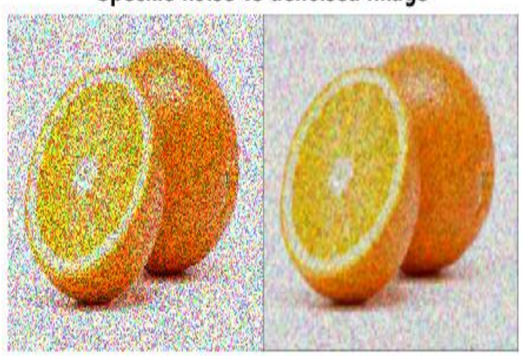
Noise reduction is usually the first process in analysis of digital images. In any image denoising algorithm it is very important that the denoising process has no blurring effect on the image and makes no changes or relocation to image edges.

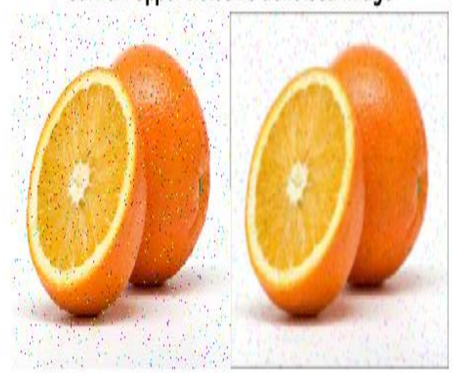
There are various methods for image denoising .Using simple filters such as average filter , Gaussian filter and median filter are some of the techniques employed for image denoising.

We have used Gaussian filter to remove the noise in the images . **Below are the MATLAB generated outputs:-**









But there is a problem that these filters reduce noise at the cost of smoothening the image and hence softening the edges .

To overcome these problems ,the partial differential equations(PDE) based method has been introduced in the literature .These methods assume that the intensity of illumination on edges varies like geometric heat flow in which heat transforms from a warm environment to a cooler one until the temperature of two environments reaches a balanced point.

The Second order PDEs :- In recent years the second order PDEs are widely used for image enhancement and denoising . **Perona and Malik** initially proposed the idea which is based on heat diffusion equations . The idea behind the use of the diffusion equation in image processing comes from the result of Gaussian filter in multiscale image analysis. Convolving the given image with a Gaussian filter k_σ:

$$k_\sigma(x, y) = \frac{1}{2\pi\sigma^2} exp(-\frac{x^2 + y^2}{2\sigma^2}) \qquad (1)$$

with standard deviation σ, is equivalent to the solution of the diffusion equation in two dimensions. Therefore, for an $N \times N$ image , the diffusion process is defined as:

$$\frac{\partial}{\partial t} I(x, y, t) = \nabla^2 I(x, y, t) = \frac{\partial}{\partial x^2} I(x, y, t) + \frac{\partial}{\partial y^2} I(x, y, t) \qquad (2)$$

$$0 \le x, y \le N, 0 \le I(x, y) \le L$$

Where L is the maximal gray level, $I(x, y, t)$ is the image $I(x, y)$ at time $t = 0.5\ \sigma^2$ with initial condition $I(x, y, 0) = I_0(x, y)$ and I_0 denotes the original image.

In general, this can be written as:

$$\frac{\partial I(x,y,t)}{\partial t} = \nabla.\left(c(x,y,t)\nabla I(x,y,t)\right) \quad (3)$$

$$I(x,y,0) = I_{\circ}(x,y)$$

Equation (3) is considered as an efficient tool for noise removal and scale space analysis of images.

Here ∇ is the gradient operator , $c(x,y,t)$ is the diffusion factor and ∇. Is the divergence operator . If c has a constant value(independent to x,y,t), the obtained equation is called a diffusion equation with an isotropic diffusion factor. In this case all the points and edges would be smoothed as there is no difference between a pixel on an edge and other pixels . It is obvious that this is not an ideal solution . To resolve this deficiency the diffusion

$$c(x,y,t) = \frac{1}{1+\frac{|\nabla I^2|}{k^2}} \quad (4)$$

$$c(x,y,t) = \exp\left(-\frac{|\nabla I^2|}{2k^2}\right) \quad (5)$$

In these equations the diffusion factor c changes at different points in the image . For those points where the gradient of the image is large, this factor has a small values . Consequently, the diffusion factor would be small around the edges , hence the edges are preserved from smoothing.

In both (4) and (5) , k is used to control the diffusion factor.

3.4 Deblurring

When we use a camera , we want the recorded image to be a faithful representation of the scene that we see-but every image is more or less blurry . Thus , image deblurring is

fundamental in making pictures sharp and useful .

In image deblurring , we seek to recover the original , sharp image by using a mathematical model of the blurring process .

In this report , we have used the inbuilt Wiener filter present in MATLAB to deblur the blurred image.

Here is the MATLAB generated output of deblurred image

4. Conclusion

PDE appear as a natural way to smooth images. When it is linear, a PDE (or equivalently the convolution) do not preserve edges. Nonlinearity is needed to preserve discontinuities (seen in all formulations) PDE may or may not derive from an optimisation problem. The notion of time evolution can be related to a notion of scale (in image restoration) but also to different aspects like a motion (in level-sets) (not shown here) Many theoretical results allow to prove if your problem is well defined or not. Giving formulations in a continuous setting offers high intuitions and discretisation aspects only come when simulations are needed. In many applications computers analyse images or image sequences which are often contaminated by noise, and their quality can be poor (e.g. in medical imaging). The models are based on the well-known Perona-Malik image selective smoothing equation and on geometrical equations of mean curvature flow type.

Convergence of the schemes to variational solutions of these strongly nonlinear problems and the extension of the methods to adaptive scheme strategies improving computational efficiency are discussed. Computational results with artificial and real 2D, 3D images and

image sequences are presented.Image Processing using PDE is a vast topic of research to help medicos in diagnosing diseases and research on complex parts of human body for example – quantification of green fluorescent protein expression in live cells(ProXcell),calculation of performance parameter of gamma cameras SPECT systems,analysis of Islet cells using automated image analysis ,working on DICOM images .

We expect that someday learning based PDEs, in their improved formulations, could be a general framework for designing PDEs for most image processing problems. The integrated system works for various applications such as video tracking, medical image processing and facial image processing. Experimental results on this applications are provided in the dissertation. Efficient computations such as multi-scale computing and parallel computing using graphic processors are also presented.

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SRI VENKATESWARA INTERNSHIP PROGRAM FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

Project Report - 2020

"Differential Equations and Mathematical Modelling"

Sri Venkateswara College
University of Delhi
Dhaula Kuan
New Delhi -110021

SRI VENKATESWARA INTERNSHIP PROGRAM FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

Project Report - 2020

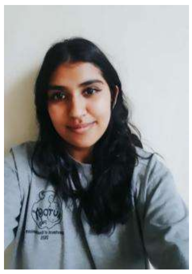
"Infectious Disease Modeling"

Sri Venkateswara College
University of Delhi
Dhaula Kuan
New Delhi -110021

SRI-VIPRA PROJECT 2020

Name of Mentor: Dr. Swarn Singh **Name of Department:** Mathematics **Designation:** Associate Professor	

Title: ***Infectious Diseases Modeling***

S.No	Name of the student	Course	Photo
1	Bhavini Malhotra	B.Sc. (H) Statistics	

Signature of Coordinator
SRIVIPRA 2020

Signature of Mentor

SRI VENKATESWARA INTERNSHIP PROGRAM FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

Project Report -2020

“Graph Theory: Problems & Applications”

Sri Venkateswara College

University of Delhi

Dhaula Kuan

New Delhi -110021

Name of Mentor: Dr. Deepti Jain **Name of Department: Mathematics** **Designation: Assistant Professor** **Sri Venkateswara College** **University of Delhi, Delhi**	

SRI-VIPRA PROJECT 2020

Title- Graph Theory: Problems & Applications

S.No	Name of the student	Course	Photo
1.	Sejal Arora (Roll Number – 1719132)	B.Sc. (Honours) Mathematics (Semester – 2)	

Signature of Coordinator
SRI-VIPRA 2020

Signature of Mentor

SRI VENKATESWARA INTERNSHIP PROGRAM FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

Project Report -2020

"SIR (D) MODEL + SIR(DV) STRUCTURE: Predictive Mathematical Modelling"

Sri Venkateswara College

University of Delhi

Dhaula Kuan

New Delhi -110021

SRIVIPRA PROJECT 2020

Title : SIR(D)MODEL+SIR(DV)STRUCTURE:Predictive Mathematical Modelling

Name of Mentor: **Dr.Swarn Singh** **Name of Department:** **Department of Mathematics** **Designation:Associate Professor**	

List of students under the SRIVIPRA Project

S.No	**Name of the student**	**Course**	**Photo**
1	Kashish	BSc(Hons) Mathematics Semester 2	
2	Dolly Goel	BSc.(Hons) Mathematics Semester 2	
3	Shriya Koul	BSc.(Hons) Mathematics Semester 2	

Signature of Coordinator **Signature of Mentor**

SRIVIPRA 2020

Certificate

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project entitled"**SIR(D) +SIRD(V) Predictive Mathematical Modelling"**.The participants have carried out the research project work under my guidance and supervision from June 15 2020 to June 30 2020. The work carried out is original and carried out in an online mode.

Signature of Mentor

Dr. Swarn Singh

Acknowledgements

We would like to express our special thanks of gratitude to our team mentor Dr.Swarn Singh for the able guidance and support in completing our project and report making. I would like to extend my gratitude to the Principal of Sri Venkateswara College Dr.S Venkata Kumar for providing us Sri VIPRA internship that was required for the welfare of students.

1st July 2020 Kashish Arora,Dolly Goel,Shriya Koul

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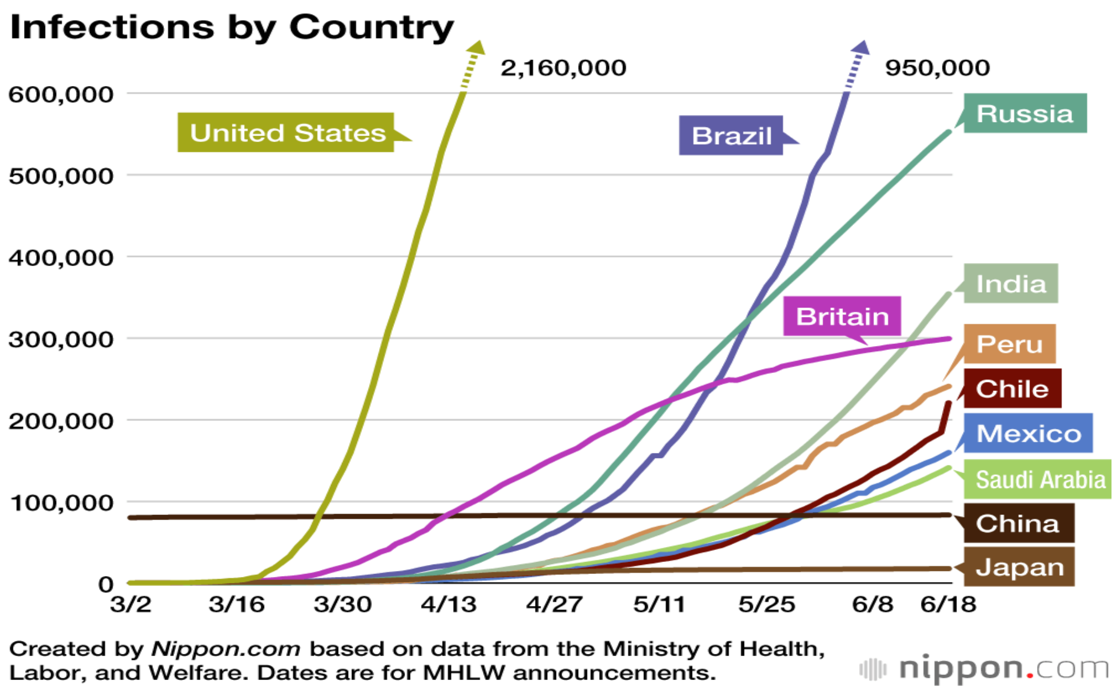
Abstract

The novel Coronavirus pathogen Covid-19 is a cause of concern across the world as the human-to-human infection caused by it is spreading at a fast pace. The virus that first manifested in Wuhan, China has travelled across continents. The increase in number of deaths in Italy, Iran, USA, and other countries has alarmed both the developed and developing countries. Scientists are working hard to develop a vaccine against the virus, but until now no breakthrough has been achieved. India, the second most populated country in the world, is working hard in all dimensions to stop the spread of community infection. Health care facilities are being updated; medical and paramedical staffs are getting trained, and many agencies are raising awareness on the issues related to this virus and its transmission. The administration is leaving no stone unturned to prepare the country to mitigate the adverse effects. However, as the number of infected patients, and those getting cured is changing differently in different states everyday it is difficult to predict the spread of the virus and its fate in Indian context. Different states have adopted measures to stop the community spread. Considering the vast size of the country, the population size and other socio-economic conditions of the states, a single uniform policy may not work to contain the disease. In this paper, we discuss a predictive mathematical model that can give us some idea of the fate of the virus, an indicative data to understand the further course this pandemic can take. Though the model is preliminary, it can be used at regional level to manage the health care system in the present scenario. The recommendations can be made, and advisories prepared based on the predictive results that can be implemented at regional levels.



1. Goal of the study

1.1 To find the number of cases at any time 't'

1.2 To find the maximum number of infectives

1.3 To find how much population can be affected

1.4 To find some factors on which the rate of pandemic depends

1.5 To find out the effect of vaccine on the number of cases

2. Introduction

The epidemiological data is up to 25th June, 2020 for India to predict Covid -19 outbreak with our SIR(D) Model +SIR(DV) Structure .The basic reproduction number R0 of India is calculated was 1.23, down from 1.29 between April 13 and May 4 and from Sitabhra Sinha, a professor at the Institute of Mathematical Sciences in Chennai. This means that between March 4 and April 11, at an R of 1.83, if 25 people had had Covid-19, they would have transmitted it to about 45 people while they were infectious. At the current R of 1.23, 25 people would infect 30, on average. Our model indicatates the number of cases at time 't'. The total number of infectives by how much population can be affected. To find some factors on which the rate of pandemic depends and the effect of vaccine on the number of cases. The attack rate or transmissibility (how rapidly the disease spreads) of a virus is indicated by its reproductive number (Ro, pronounced R-nought or r-zero), which represents the average number of people to which a single infected person will transmit the virus.WHO's estimated (on Jan. 23) Ro to be between 6/26/2020 3 1.4 and 2.5. Other studies have estimated a Ro between 3.6 and 4.0, and between 2.24 to 3.58.Preliminary studies had estimated Ro to be between 1.5 and 3.5. [An outbreak with a reproductive number of below 1 will gradually disappear .For comparison, the Ro for the common flu is 1.3 and for SARS it was 2.0.]

3. Model Assumption

3.1 We assume that population of Susceptible and Contagious are large,so that the random differences between individuals can be neglected .(Ignoring differences of the kind of immune system everybody has.

3.2 Recovered patients are supposed to immune to Covid-19.

3.3 Susceptibles and Infected ones are evenly spread.

3.4 Ignoring the Emmigrants and Immigrants.

3.5 Assuming all the constants to be same throughout .

3.6 Ignoring natural Birth and Death Rates. (Meaning that population fluctuation is being neglected).

3.7 Natural Death is ignored after vaccination .

3.8 Initially taking recovered and dead as 0 .

3.9 Days of recovery and death is taken as 14

4. Time Dependent Variables

4.1 S(t):-The people other than infectives and recovered are suspected to get the disease .That's why they are known as susceptibles. (Susceptibles at any time 't'.)

4.2 I(t):-Number of infected people from Covid-19 at any time 't'.

4.3 R(t) :- Number of recovered patients from Covid-19 at any time 't'

4.4 D(t) :- Number of deaths at any time 't' from Covid – 19

5.Rate Constants

5.1 P:-Total population (taken constant).

5.2 λ(t) :- Variable constant rate for vaccination in the number of susceptibles. It depends upon the 4 factors:-

5.2.1 Number of Infectives – I(t) or we can say ratio of I(t) to total population ,(I[t]:P)

5.2.2 Number of contacts – c (per time)

5.2.3 Probability that a contact resut in an infection – p.

5.2.4 Latent period (time after which the infected can spread the disease-l

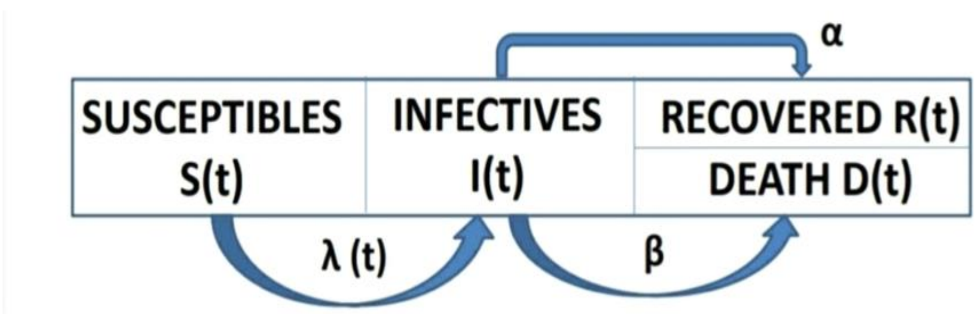
5.3 α:- Rate constant to define the variations in recovered patients.

α^{-1} :- It is defined as the time period of an individual speeds as infective.

5.4 β:- Rate constant to define the variation in dead patients

6.Making of the model

Our first model is SIR(D) model which consist of three compartment levels : Susceptible (S), Infectious (I) Recovered (R) and Death (D). S represents the people other than infectives and recovered are suspected to get the disease. I are the number of infected people from COVID -19 at any time 't'. R are the number of recovered patients from Covid - 19 at any time 't'. D are the number of dead patients from Covid - 19 at any time 't'. Also, it assumes that within the outbreak period, no significance population change takes place (e.g., through new births, deaths, migration etc.) and P = S + I + R +D = Constant.Also, we assume many more assumptions i.e. Recovered patients are supposed to immune to Covid - 19. Susceptibles and infected ones are evenly spred. Ignoring the emmingrents and immigrants. All the constants to be same throughout. Natural dead is ignored after vaccination. Intially, taking dead and recovered as 0 and days of recovery and number of deaths is taken to be constant.



Our SIR(D) model can be expressed by the following set of differential equations as follows :-

6.1 S (t) – Susceptible

$$dS/dt \propto S$$

$$dS/dt = -\lambda(t)\, S(t)$$

[Negative sign because susceptible population keeps on decreasing and gets converted into infectives.]

where λ(t) is a variable constant rate for in no. of susceptibles. It also depends upon 4 factors :- infectives I(t),contacts (c),probability of getting infection (p) and latent period (ℓ).

$$\lambda(t) = c \times p \times I(t)/P \times \ell$$

$c * p * \ell =$ **B_f (Transmission Constant)**

$$\lambda(t) = Bf * I(t)/P$$

∴ dS/dt = - Bf * I(t)/P * S(t) - Equation – 1

S(0) = S0

where P is the total population (taking constant).

6.2 I (t) – Infectives

Rate of change in number of infectives = rate of susceptibles infected – rate of infected recovered – rate of infective died

$$dI/dt \propto I(t),\ R(t),\ D(t),\ S(t)$$

$$\therefore\ dI/dt = \lambda(t) \times S(t) - \alpha \times I(t) - \beta \times I(t)$$

where α is the rate constant to define the variations in recovered patients and β is the rate constant to define the variations in dead patients.

$dI/dt = Bf * I(t)/P * S(t) - \alpha \times I(t) - \beta \times I(t)$ $I(0) = I_0$

∴ dI/dt = I(t) [B_f * S(t)/P – α – β] - <u>Equation 2</u>

6.3 R(t) – Recovered

Rate of change in number of recovered is directly proportional to rate of infectives

$dR/dt \propto I(t) \quad R(0) = R_0$

∴ $dR/dt = \alpha \times I(t)$ - Equation - 3

6.4 D(t) – Dead

Rate of change in number of dead is directly proportional to rate of infectives

$dD/dt \propto I(t) \quad D(0) = D_0$

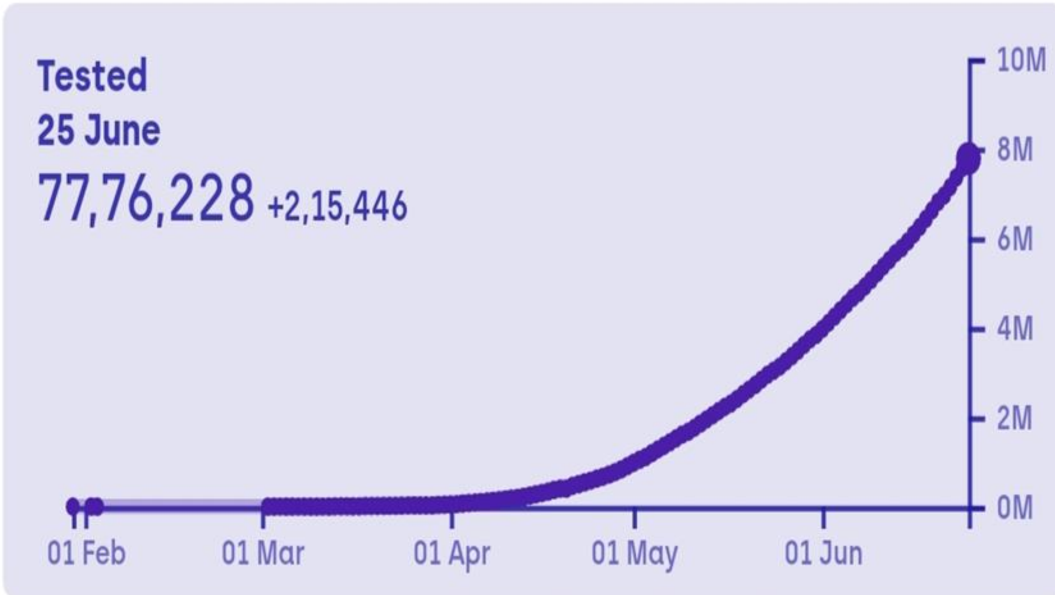
∴ $dD/dt = \beta \times I(t)$ - Equation - 4

6.5 Determining the value of constants for India

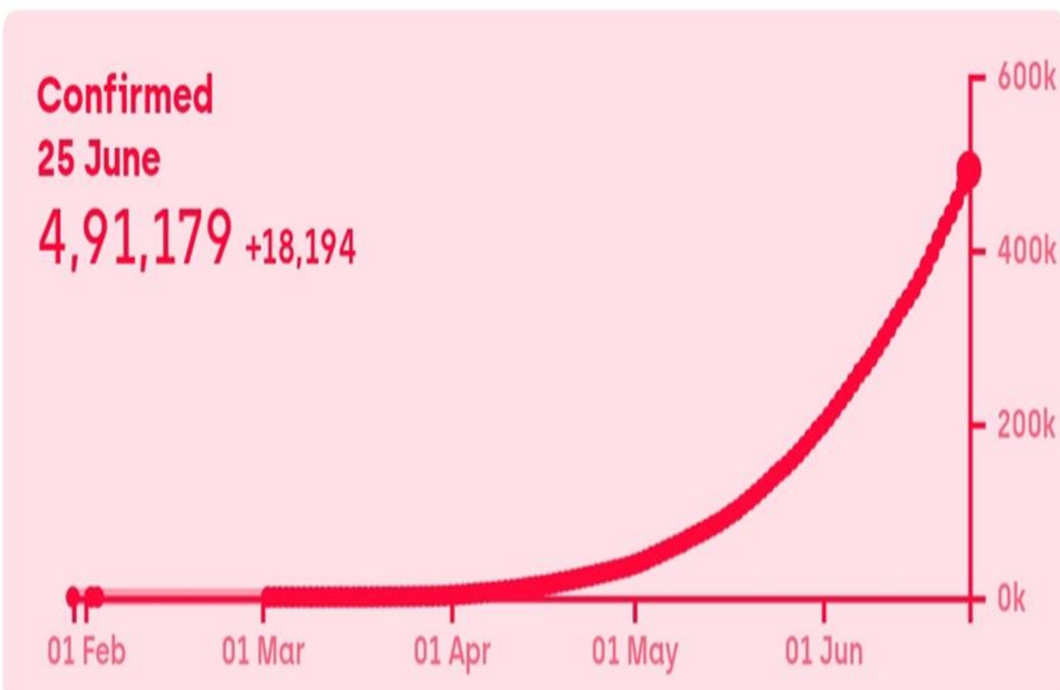
- For λ(t) - c = 11 (contact per person) , p = 0.8 (transmission), ℓ = 3 days, P = 1, Taking these in consideration and the proportion of people that go for testing. Bf = 0.0987 .
- For α (recovery constant) - Depends upon a) % of recovered and b) number of days for recovery (α = 55.6 % / 14 days = 0.039714)
- For β - Depends upon no. of days for death (14) (β = 5 days / 14 days = 0.0035714)
- I0 = 108 *10(-9) (as of 15th March)
- S0 = 0.999
- R(0) = 0
- D(0) = 0

6.6 Actual Graph till 25th June

TOTAL CASES TESTED IN INDIA :-



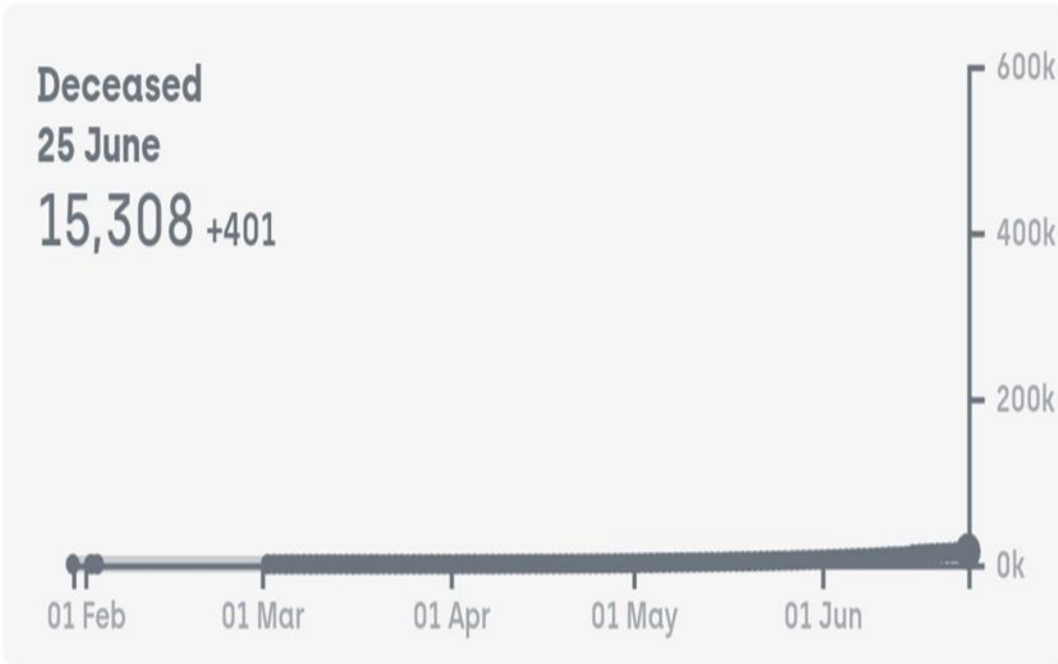
CONFIRMED CASES IN INDIA :-



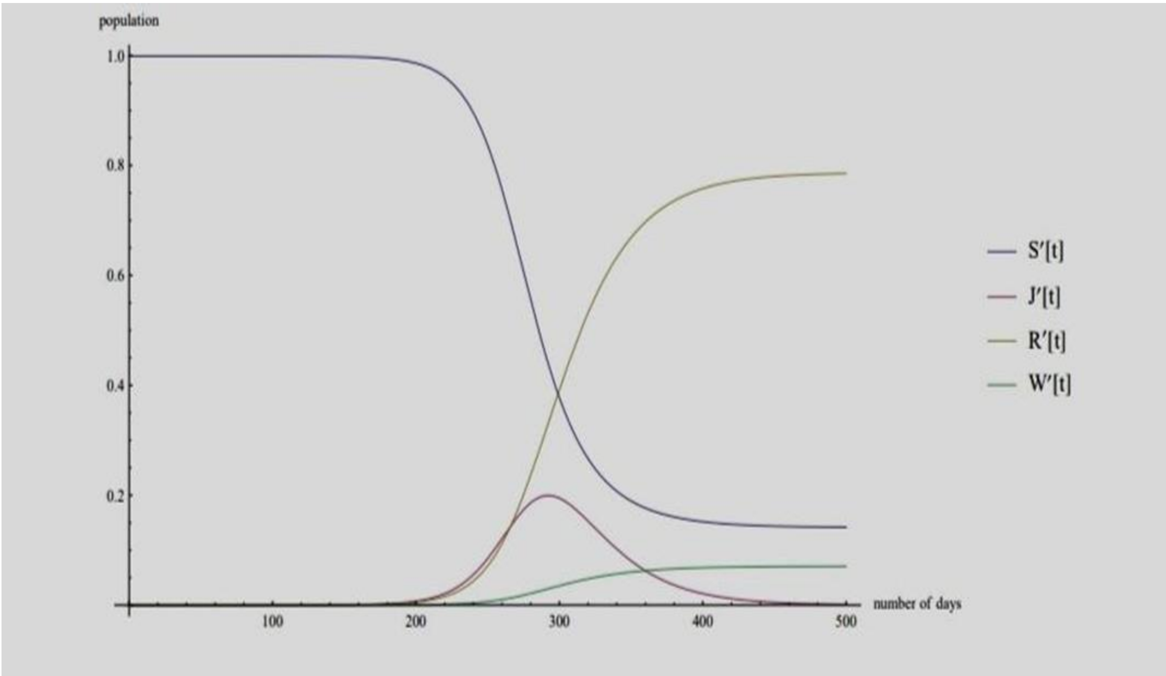
ACTIVE CASES IN INDIA :-

RECOVERED CASES IN INDIA :-

DEAD CASES IN INDIA :-



6.6 Graph that our model depicts(500 days after 15th March)



- Our graph represents drastic decrease in the number of susceptibles .
- The number of infectives increase exponentially then decrease after affecting a large number of population .

- The number of recovered people seems to increase .
- The number of dead patients seems to be constant .
- According to our model the number of infectives will reach at its maximum values after about 350 days .

7. Determining the factors of disease spread

So, to determine the growth of a pandemic, we need to check the rate at which the infectives increasing.

$$\text{As}\ ,dI/dt = I(t)\ [B_f * S(t)/P - \alpha - \beta\ ]$$

As, we had to determine the growth of a pandemic, so from the above equation we clearly see that if **$[B_f * S(t)/P - \alpha - \beta\ ]$** is decreasing , then the growth of pandemic is also going down and vice- versa.

$$\text{as}\quad \mathbf{dS/dt = - B_f * S(t)/P * I(t)}$$

Since S(t), B_f/P, I(t) all are constants. So, S(t) is also going to decrease due to negative sign

$$\mathbf{S \leq So}\ \text{(as the rate of S is going negative)}$$

$$\mathbf{dS/dt < I(t) \times [\ B_f/P \times S_0 - \alpha - \beta\ ]}$$

now, to determine the factor we was taking **$B_f/P = x$**.

dI/dt = I(t) [x S(t) – α – β]

So if(x .S_0 –α –β) is positive then there will be a spread of the disease.

$$\text{If , } x .S_0 - \alpha - \beta > 0$$

$$x .S_0 > \alpha+\beta$$

$$S_0 > (\alpha+\beta) /x$$

$$\Rightarrow\quad S_0 . x /(\alpha+\beta) > 1$$

$$\text{Let } (\alpha+\beta) / x = 1/y$$

$$\Rightarrow\quad \mathbf{S_0 * y > 1}$$

$$\text{Let } \mathbf{R_0 = S_0 . x/(\alpha+\beta) = S_0 \times y}$$

7.1 Basic Reproductive ratio

R_0 as reproductive ratio that defines the rate at which infectives spread the disease through their contacts.

$$\text{If } R_0 > 1 \quad (S_0 \times y > 1)$$

Then the disease is likely to spread and converted into a pandemic.

There are 3 cases :-

1. **$R_0 < 1$**.

The outbreak will become **extinct**. Since this implies that he infected can spread the virus to less than one person.

2. **$R_0 = 1$**.

If R_0 is equal to 1, then one infected person will infect exactly one other person, and so the number of infected persons in the population will remain **constant** over time.

3. **R_0 > 1**.

Then outbreak will lead to an **epidemic**. For Covid-19 the value of basic reproductive ratio is around 3 (in world) and 2.56 in India, which is greater than 1 and clearly not a good sign for the outbreak.

8. Maximum number of infectives

For finding out the maximum number of infectives we need to find out dI/dS and then we will put that equal to 0 to find out the maxima

$$\mathbf{dS/dt = -Bf/P \times I(t) \times S(t)}$$

$$\mathbf{dI/dt = I(t) \times [Bf/P \times S(t) - \alpha - \beta]}$$

Finding, dI/dS

$$\mathbf{(dI/dt) \times (dt/dS) = I(t) \times [Bf/P \cdot S(t) - \alpha - \beta]}$$

$$\mathbf{I(t) \times [-Bf/P \times S(t)]}$$

$$\mathbf{dI/dS = -1 + [(\alpha+\beta)/Bf] \times P/S(t)}$$

$$\mathbf{Bf/P = x} \text{ and } \mathbf{(\alpha+\beta)/x = 1/y}$$

$$dI/dS = -1 + [1/y \times S(t)]$$

$$\mathbf{dI = -1 + [1/y \times S(t)] \times dS}$$

Integrate dI/dS , we get

$$\mathbf{I = -S + \ell n\, S/y + c}$$

At, **t =0** , **I(0)=I_0** and **S(0)=S_0**

$$c = I_0 + S_0 - \ell n S_0/y$$

$$\mathbf{I = -S + \ell n S/y + I_0 + S_0 - \ell n S_0/y}$$

$$\mathbf{I + S - \ell n S/y = I_0 + S_0 - \ell n S_0/y}$$

To find out the maximum value we put ,

$$\mathbf{dI/dS = 1 - (1/y) \times S(t) = 0.}$$

$$\therefore \quad \mathbf{1 = 1/[y \times S(t)]}$$

$$\mathbf{S(t) = 1/y}$$

$\therefore$ I will be maximum when **S(t) = 1/y**

I_{max} comes out to be equal to -

$$I_{max} + 1/y - 1/y \times \ell n(1/y) = I_0 + S_0 - \ell n S_0 / y$$

$I_{max} = I_0 + So + (1/y) \times \ell\, n(1/y) – \ell n S_0 / y - 1/y$

$$I_{max} = I_0 + S_0 – (1/y) \times [\ell\, n(y \times S_0) + 1]$$

Since $I_0 + S_0$ is the total population, the maximum value of I_0 depends on the factor – **[ℓ n(y × S₀) +1] / y**.

1/y is defined as **(α+β)/x** and **x= B_f / P**

$$\therefore 1/y = (\alpha+\beta) \times P/B_f$$

Since, y is the contact ratio and in this case of Covid-19, y is quite large due to high rate of getting infected by the infectives.

⇒ 1/y is very small

Making $1/y \times [\ell\, n(y \times S_0)+1]$ small.

Since, $I_{max} = I_0 + S_0 - 1/y \times [\ell\, n(y \times S_o)+1]$

⇒ Implies majority of population will get infected.

9. Effect of vaccine

Let h be the number of people vaccined per day.

∴ Therefore , number of people vaccined in t days will be h×t.

So, if we start vaccinating people, then the number of susceptible will decrease and those will convert into recovered.

∴ The final equation after the vaccination becomes available.

1. **dS/dt = - B_f / P × I(t) × S(t) – h × t**
2. **dI/dt = I(t) × [B_f / P – α – β – h × t]**
3. **dR/dt = α × I(t) + h × t**
4. **dD/dt = β × I(t)**

10. Conclusion

- The number of Covid-19 cases at any time depends upon the susceptibles ,infectives, average contact , probability of transmission , latent period , recovery rate , death rate and population density**.**
- The growth of pandemic depends upon basic reproductive factor R_0, which is the number of people an infected individual can affect. S_0 to flatten the number of cases, the value of R_0 must be less than 1
- The maximum number of infectives at any time is majority of the population , since the transmission rate is very high in case if Covid -19.
- The maximum number of infectives at any time is majority of the population , since the transmission rate is very high in case if Covid -19.

11. References

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