SRI VENKATESWARA INTERNSHIP PROGRAM FOR RESEARCH IN ACADEMICS (SRI-VIPRA)

SRI-VIPRA

Project Report of 2023: SVP-2302

"APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS"

IQAC
Sri Venkateswara College
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SRIVIPRA PROJECT 2023

Title : Applications of Partial Differential Equations

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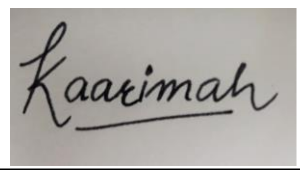
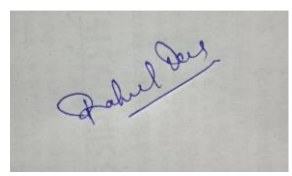
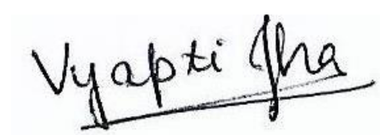
1		Kaarimah	1721089	B.Sc. (H) Mathematics	
2		Rahul Das	1721041	B.Sc. (H) Mathematics	
3		Vyapti Jha	1721061	B.Sc.(Hons) Mathematics	
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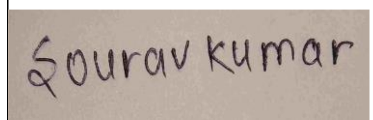
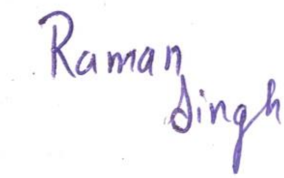
6		Sourav Kumar	1721026	B.Sc.(Hons) Mathematics	
7		Raman Singh	1721073	B.Sc. (Hons.) Mathematics	
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Signature of Mentor

Certificate of Originality

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project **SVP-2302** titled "**Application of Partial Differential Equations"**. The participants have carried out the research project work under my guidance and supervision from 15 June, 2023 to 15th September 2023. The work carried out is original and carried out in an online/offline/hybrid mode.

Signature of Mentor

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We would like to thank the entire faculty of Mathematics Department who directly or indirectly helped us to complete this project report and finally we extend our sincere thanks to SRIVIPRA-2023 for initiating such a research platform.

TABLE OF CONTENTS

EXACT SOLUTION OF A CASSON FLUID FLOW INDUCED BY DUST PARTICLES WITH HYBRID NANOFLUID OVER A STRETCHING SHEET SUBJECT

Kaarimah and Rahul Das

Flow of Casson hybrid fluid in a stretched sheet induced by dust particles is studied. The equations governing the fluid flow are given as below.

Fluid phase Momentum equation:

Momentum Equation:

$$\rho_{\text{hbnf}}\left(u(x,y)\frac{\partial u(x,y)}{\partial x}+v(x,y)\frac{\partial u(x,y)}{\partial y}\right)$$
$$=\mu_{\text{hbnf}}\left(1+\frac{1}{\gamma_A}\right)\frac{\partial^2 u(x,y)}{\partial x^2}-\sigma_{\text{hbnf}}B_0^2\mu$$
$$+K_A N_A\left(u_d(x,y)-u(x,y)\right)-\frac{\gamma}{K_p\mu}$$

Dust phase continuity equation:

$$\textbf{Continuity Equation:}\quad \frac{\partial u_d(x,y)}{\partial x}+\frac{\partial v_d(x,y)}{\partial y}$$

Dust phase momentum equation:

Momentum Equation:

$$m_A\left(u_d(x,y)\frac{\partial u_d(x,y)}{\partial x}+v_d(x,y)\frac{\partial u_d(x,y)}{\partial y}\right)$$
$$=K_A\left(-u_d(x,y)+u(x,y)\right)$$

Boundary conditions:

Fluid phase: $u(x,0)=u_w(x),\quad v(x,0)=0$

$u(x,y)\to 0,\quad u_d(x,y)\to 0,\quad v_d(x,y)\to v\quad \text{as } y\to\infty$

The above equations are solved analytically by the method of transformation to get the solutions for velocity for flow of fluid and dust particles.

The impact of the parameters βvA, MA and γA on fluid and dust velocity are observed graphically. The graphs are plotted using software Mathematica. From the graph following observations are made:

- Increase in fluid particle interaction (βvA) doesn’t have much impact on fluid velocity but adrastic increase in dust velocity is noticed.
- As Casson parameter (γA) increases there is decrease in velocity of both fluid and dust.
- Increase in magnetic parameter decreases velocity.

CLOSED FORM SOLUTION OF A JEFFREY FLUID FLOW INDUCED BY DUST PARTICLES WITH HYBRID NANOFLUID OVER A STRETCHED SHEET

Vyapti Jha and Ananya Malhotra

We use the following constants in the equations for fluid phase and dust phase: K_A, *the Stokes drag coefficient*; N_A, the dust particle number; m_A, the mass of the dust particle; λ_1, the non-Newtonian *Jeffrey parameter*. Other symbols used such as μ_{hbnf}, ρ_{hbnf} and σ_{hbnf} denotes absolute viscosity, density of hybrid nano-fluid and electrical conductivity of hybrid nano-fluid respectively. Here, B_0 is the uniform magnetic field.

Keeping in mind the assumptions, we consider equations for the fluid phase and dust phase for the hybrid-nanofluid with the boundary conditions as follows:

Fluid phase:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} = 0$$

$$\rho_{hbnf}\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial y}\right) = \mu_{hbnf}\left(1+\frac{1}{\lambda 1}\right)\frac{\partial^2 u}{\partial y^2} - \sigma_{hbnf}B_0^2 u + K_A N_A (u_d - u)$$

,

Dust phase:

$$\frac{\partial u_d}{\partial x} + \frac{\partial w_d}{\partial y} = 0$$

$$m_A\left(u_d\frac{\partial u_d}{\partial x} + w_d\frac{\partial u_d}{\partial y}\right) = K_A(u - u_d)$$

which are subject to following boundary conditions

$$\begin{cases} u = u_w(x), w = 0, \quad at \quad y = 0, \\ u \to 0, u_d \to 0, w_d \to \quad as \quad y \to \infty \end{cases}$$

Thus, we have the following exact solutions:

Fluid phase:

$$F(\xi) = \frac{1}{Z_A} - \frac{1}{Z_A}e^{-Z_A\xi}$$

Dust phase:

$$G(\xi) = \frac{1}{Z_A} - \frac{\beta\nu_A}{Z_A(1+\beta_{\nu A})}e^{-Z_A\xi}$$

where $Z_A = \sqrt{\dfrac{\frac{\rho_{hbnf}}{\rho_f} + \frac{\sigma_{hbnf}}{\sigma_f}M_A + \frac{L_A\beta_{\nu A}}{1+\beta_{\nu A}}}{\frac{\mu_{hbnf}}{\mu_f}\left(1+\frac{1}{\lambda_1}\right)}}$

PERISTALTIC TRANSPORT OF JEFFREY FLUID IN AN ELASTIC TUBE

Ankita Chakraborty

In this work I took the peristaltic flow of a steady non-compressible Jeffrey fluid in an elastic tube of length L and radius α (z). The tube walls are subjected to an infinite sinusoidal wave movement with the constant speed c. At any axial station z, the instantaneous radius of the tube is given by,

$$\bar{R} = \bar{\alpha}'(\bar{z},\bar{t}) = \alpha_0 + b\, sin\frac{2\pi}{\lambda}(\bar{Z} - c\bar{t})$$

The governing equations and its solution of the fluid flow under certain assumptions are represented below.

$$\frac{\partial}{\partial r}\left[\mathrm{r}\left\{\frac{1}{1+\lambda_1}\left(-\frac{\partial w}{\partial r}\right)\right\}\right] = -\mathrm{r}\,\frac{\partial p}{\partial z}$$

$$F = K\,[f(\alpha''_1) - f(\alpha''_2)] - 1 - \frac{\varphi^2}{2} - \frac{\varphi}{\pi}$$

Where,

$$f(\alpha'') = -\frac{2}{7}(t_2 + 7t_1)\alpha' + 4(t_1 + t_2)\alpha'^3 log\alpha'' - 2(t_2 + 3t_1)\alpha'^2 + 4t_2\alpha'^4 + \frac{1073}{210}t_2 - \frac{t_1}{3} - \frac{(t_1+t_2)\alpha'^4}{\alpha''} + [6(t_1 + t_2)\alpha'^2 - 10t_2\alpha'^4\ ]\ \ \alpha''^2 + \left[2(t_1 + t_2)\alpha' - 20t_2\alpha'^3 + 10t_2\alpha'^4\right]\alpha''^2 + \frac{1}{3}\left(t_1 + t_2 - 60t_2\alpha'^2 + 80t_2\alpha'^3 - 15t_2\alpha'^4\right)\alpha''^3 + t_2\alpha'\left(30\alpha' - 15\alpha'^2 + \alpha'^3 - 10\right)\alpha''^4 + 2t_2\left(8\alpha' - 9\alpha'^2 + \frac{8}{5}\alpha'^3 - 1\right)\alpha''^5 + t_2\left(\frac{10}{3} - 10\alpha' + 4\alpha'^2\right)\alpha''^6 + \frac{t_2}{7}(16\alpha' - 15)\alpha''^7 + \frac{1}{2}t_2\alpha''^8$$

In the present study, the flow of a Jeffrey fluid through elastic tube with peristalsis has been investigated. The effects of various parameters like amplitude ratio, elastic parameters t_1 and t_2, inlet and outlate elastic radius on volume flow rate F are discussed graphically.

We have analyzed the variation of flux for different values of amplitude ratio ϕ = 0.4, 0.5, 0.6, observed that there is an increase in flux with the increase in ϕ. The change in volume flow rate of a Jeffrey fluid in an elastic tube for different values of inlet and outlet radius was noticed. Flux decreases for increasing values of inlet radius and the opposite behavior is noticed with outlet radius. Also, the flux increases with increasing values of both elastic parameters t_1 = 5, 10, 15 and t_2 = 100, 200, 300.

CLOSED FORM SOLUTION OF ELECTROMAGNETIC FORCE ON NANOFLUID FLOW: A COMPARISON

Sourav Kumar

We have considered an unsteady two-dimensional flow heat transfer of a nanofluid compressed in the middle of two parallel plates infinitely extended and implanted system with nanofluid (water as base fluid) contains various nanoparticles, ie. copper (Cu), silver (Ag), aluminum oxide (Al2O3) and titanium oxide (T iO2) with the sliding speed effect. Thermophysical properties nanofluids are given in Table 1. ”Transverse magnetic field” variable intensity is set diagonally to both discs. The distance between the two plates is z = 2H p (1 − at) where H is the initial position (at point t = 0). Akbari Ganji method is used to find analytical solution of nanofluid with electromagnetic force. Two different models are compared: one is Casson nanofluid and other is nanofluid with permeability. The momentum and energy expressions in both the cases are

Casson nanofluid

$$f(\eta) = -0.0423\eta^5 + 0.4154\eta^3 + 1.4577\eta$$
$$\theta(\eta) = 0.1902\eta^3 + 0.2699\eta^2 + 0.5399$$

Nanofluid with permeability

$$f(\eta) = 4.6645\eta^5 - 9.8289\eta^3 + 6.1645\eta$$
$$\theta(\eta) = 0.1902\eta^3 + 0.2699\eta^2 + 0.5399$$

The Impact of Squeeze number (Sq), Hartmann number (Ha), Permeability Parameter (kp) casson parameter (β) on vertical velocity are observed graphically. The following conclusions are observed:

- Increase in Squeeze number results in velocity decrease for casson nanofluid and in presence of permeability.
- It is also observed that as the squeeze number is less there is a lot of change in velocities between both models and as Squeeze number is more, the difference between the velocities are very less.
- Velocity of nanofluid in presence of permeability is more for least value of Sq.
- More the value of Ha, the less is the velocity in both the models.
- Also at Ha=1, the difference between both the velocities are observed clearly. Velocity of Casson nanofluid is more compared to velocity of nanofluid in presence of permeability.
- As the Casson parameter increases upto 1 the velocity decreases, but after 1 the velocity increase with that of Casson parameter.
- Permeability parameter (kp) and velocity of nanofluid with permeability travel in the opposite direction.

FLOW OF CASSON FLUID WITHIN A CHANNEL OF ELASTIC WALLS

Raman Singh and Nishant Gupta

We Considered the steady, laminar, incompressible, Poiseuille flow of a Casson fluid in a horizontal channel with elastic plane analogue L and $an(x)$ be the length and half- width of the channel respectively, μ is the viscosity of the Herschel-Bulkley fluid and τ_{yx} is the shear stress.The region between $y = 0$ and $y = y_0$ is called a plug flow region ,$|\tau_{yx}| \leq \tau_0$. The momentum equation governing the flow are solved analytically with suitable boundary conditions to obtain the solution as

$$q = G[\int_{p_2-p_o}^{p_1-p_o} \frac{a^3 t_1}{a^2} + \left(4a^3 - 15a^2 + \frac{1}{a^2} + 20a - 10\right) t_2]da$$

$$q = G[g(a_1) - g(a_2)]$$

$$g(a) = \frac{a^2 t_1}{2} + \left(\frac{4a^7}{7} - \frac{5a^6}{2} + 4a^5 - \frac{5a^4}{2} + \frac{a^2}{2}\right) t_2$$

where $a_1 = a(p_1 - p_0), a_2 = a(p_2 - p_0)$

The governing equations are solved analytically to obtain velocity, pressure and flux expressions in terms of various physical parameters. The impact of various parameters we noticed graphically by using Mathematica software.

The observations are as follows:

- It is noticed that as yield stress increases flux decreases.
- An increase in outlet pressure decreases flow rate and the opposite behaviour is noticed with inlet pressure.
- Elasticity is measured by two parameters t_1, t_2 and the parameter t_2 has a major impact on flux as it increases flux is increasing. Parameter t_1 doesn't have much impact on flow rate.