



**SRI VENKATESWARA INTERNSHIP PROGRAM
FOR RESEARCH IN ACADEMICS
(SRI-VIPRA)
Student Internship**



SRI-VIPRA


Project Report of 2025: SVP-2506

“Partial Differential Equation in Fluid Flows”




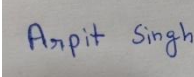

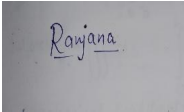


**IQAC
Sri Venkateswara College
University of Delhi
Benito Juarez Road, Dhaula Kuan, New Delhi
New Delhi -110021**

SRIVIPRA PROJECT 2025

Title : Partial Differential Equation in Fluid Flows

Name of Mentor: Dr. P. Devaki Name of Department: Mathematics Designation: Assistant Professor	
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List of students under the SRIVIPRA Project

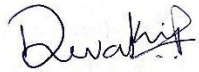
S.No	Photo	Name of the student	Roll number	Course	Signature
1		Dhruv Agrawal	1724041	B.Sc. (H) Mathematics	
2		Arpit Singh	1724047	B.Sc. (H) Mathematics	
3		Ranjana	1823005	B.Sc. (H) Physics	
4		Yatin Sood	1723067	B. Sc. (H) Mathematics	



Signature of Mentor

Certificate of Originality

This is to certify that the aforementioned students from Sri Venkateswara College have participated in the summer project SVP-2506 titled Partial Differential Equation in Fluid Flow. The participants have carried out the research project work under my guidance and supervision from 1st July, 2025 to 30th September 2025. The work carried out is original and carried out in an online/offline.



Signature of Mentor

Acknowledgements

We would like to express our profound appreciation to all those who contributed to the successful completion of our project. Our deepest gratitude goes to our esteemed supervisor, **Dr. P. Devaki**, whose invaluable guidance, expertise, and unwavering support were pivotal throughout this research journey. Her insightful feedback and encouragement inspired us to strive for excellence. Additionally, we would like to extend our gratitude to our honorable principal, **Prof. Vajala Ravi** for providing resources and facilities, which played a crucial role in facilitating our work.

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Types of Fluids

In fluid mechanics, a **fluid** is defined as a substance that deforms continuously under the action of applied shear stress, regardless of how small the stress is. Fluids are generally classified based on their physical properties, behaviour under stress, and compressibility. The major types of fluids are as follows:

1. **Ideal Fluid**

An ideal fluid is an imaginary concept used for simplification in theoretical studies. It is assumed to be incompressible and non-viscous, meaning it offers no resistance to shear stress. Since viscosity cannot be neglected in real-life applications, ideal fluids do not exist in reality but are useful for analysis.

2. **Real Fluid**

A real fluid is any practical fluid that has viscosity and compressibility. All natural fluids such as water, air, and oil are real fluids. They exhibit resistance to flow and energy loss due to friction.

3. **Newtonian Fluid**

A Newtonian fluid is one in which the shear stress is directly proportional to the rate of shear strain. The viscosity of such fluids remains constant regardless of the applied stress. Common examples include water, air, and thin oils.

4. **Non-Newtonian Fluid**

These are fluids that do not obey Newton's law of viscosity. In these fluids, the relationship between shear stress and strain rate is nonlinear, and viscosity may change with time or stress. Examples include toothpaste, blood, ketchup, and polymer solutions.

5. **Ideal Plastic Fluid**

This is a type of fluid that behaves as a solid until a certain yield stress is reached. Once the yield value is crossed, the fluid begins to flow like a liquid. Examples include toothpaste and clay suspensions.

6. **Compressible and Incompressible Fluids**

- **Compressible fluids** are those whose density changes significantly with variations in pressure and temperature. Gases are generally considered compressible.
- **Incompressible fluids** are those whose density remains nearly constant, even under high pressure. Liquids like water are commonly treated as incompressible for engineering calculations.

Types of Equations

1. Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

It represents conservation of mass in fluid flow. The mass entering a system equals the mass leaving or stored within.

2. Euler's Equation

$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho} \nabla p + \mathbf{g}$$

Describes the motion of an ideal (non-viscous) fluid. It links pressure forces, gravity, and fluid acceleration.

3. Navier–Stokes Equation

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \mu \nabla^2 \mathbf{V} + \rho \mathbf{g}$$

Governs motion of real fluids with viscosity. It is widely applied in engineering and industrial flow problems.

4. Bernoulli's Equation

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

Represents conservation of energy along a streamline. It relates pressure head, velocity head, and elevation head.

5. Energy Equation

$$\Delta E = Q - W$$

Shows conservation of total energy in a system. It considers internal, kinetic, potential energies, heat, and work.

6. Momentum Equation (Integral Form)

$$\sum \mathbf{F} = \frac{d}{dt} \int_{CV} \rho \mathbf{V} dV + \int_{CS} \rho \mathbf{V} (\mathbf{V} \cdot d\mathbf{A})$$

Relates external forces to the rate of momentum change. Used in cases like jets, nozzles, and pipe bends.

Mathematical model

Notation

- ε : amplitude ratio
 β : Casson fluid parameter
 M : Magnetic parameter
 E_1 : Tension parameter
 E_2 : Wall mass description parameter
 E_3 : Damping force parameter
 (x,y) : Cartesian coordinates
 t : time
 $h = \varepsilon \sin(2\pi(x-t)) + 1$ (Wall deformation)

Partial Differential Equation Analyzed

$$\frac{\partial^4 \psi}{\partial y^4} - N^2 \frac{\partial^2 \psi}{\partial y^2} = 0 \dots (1)$$

Where

$$N^2 = M^2 \left(1 + \frac{1}{\beta} \right)$$

General Solution of (1)

$$\psi = c_1 + c_2 y + c_3 e^{Ny} + c_4 e^{-Ny}$$

Boundary Conditions

(A) $\psi = 0$ at $y = 0$

(B) $\frac{\partial \psi}{\partial y} = 0$ at $y = h$

(C) $\frac{\partial^2 \psi}{\partial y^2} = 0$ at $y = 0$

$$\left(1 + \frac{1}{\beta}\right) \frac{\partial^3 \Psi}{\partial y^3} - M^2 \frac{\partial \Psi}{\partial y} - \left(E_3 \frac{\partial^2 h}{\partial x \partial t} + E_1 \frac{\partial^3 h}{\partial x^3} + E_2 \frac{\partial^3 h}{\partial x \partial t^2}\right) = 0 \quad (D)$$

$$\Rightarrow \left(1 + \frac{1}{\beta}\right) \frac{\partial y^3}{\partial^3 \Psi} - M^2 \frac{\partial y}{\partial \Psi} = 8\epsilon\pi^3 \left[\frac{E_3}{2\pi} \sin 2\pi(x-t) - (E_1 + E_2) \cos 2\pi(x-t) \right]$$

$$\Rightarrow \left(1 + \frac{1}{\beta}\right) \frac{\partial y^3}{\partial^3 \Psi} - M^2 \frac{\partial y}{\partial \Psi} = a$$

Where

$$a = 8\epsilon\pi^3 \left[\frac{E_3}{2\pi} \sin 2\pi(x-t) - (E_1 + E_2) \cos 2\pi(x-t) \right]$$

Applying Boundary Conditions

$$c_1 = 0, c_2 = -\frac{a}{N^2\left(1+\frac{1}{\beta}\right)}, c_3 = \frac{a}{2N^3\left(1+\frac{1}{\beta}\right)\cosh(Nh)}, c_4 = -\frac{a}{2N^3\left(1+\frac{1}{\beta}\right)\cosh(Nh)}.$$

Solution

$$\Psi = -\frac{ay}{N^2\left(1+\frac{1}{\beta}\right)} + \frac{ae^{Ny}}{2N^3\left(1+\frac{1}{\beta}\right)\cos(Nh)} - \frac{ae^{-Ny}}{2N^3\left(1+\frac{1}{\beta}\right)\cos(Nh)}$$

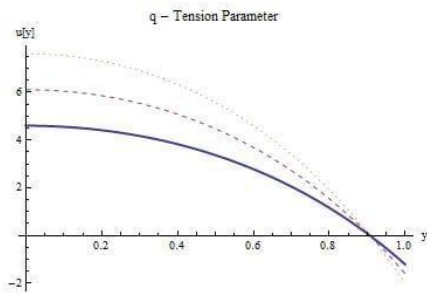
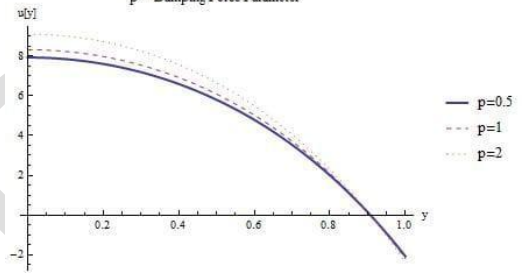
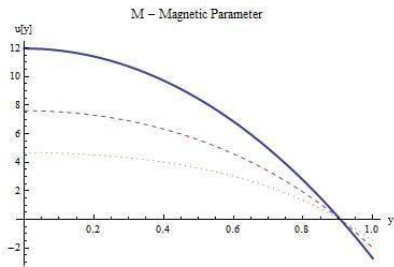
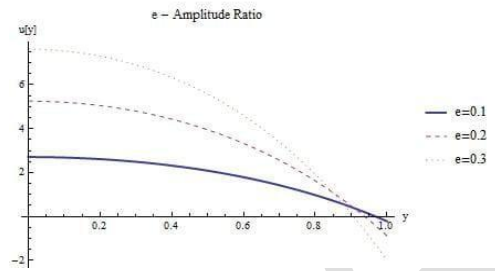
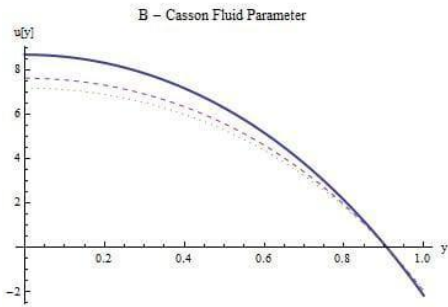
$$\Psi = -\frac{ay}{N^2\left(1+\frac{1}{\beta}\right)} + \frac{a\sinh(Ny)}{N^3\left(1+\frac{1}{\beta}\right)\cosh(Nh)}.$$

Newtonian Fluid Case

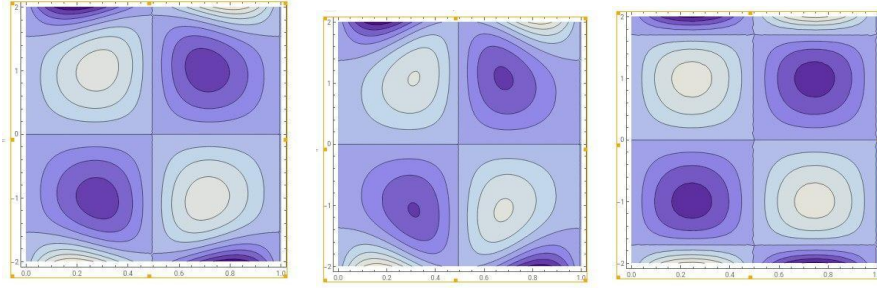
For a Newtonian fluid ($\beta \rightarrow \infty$):

$$\Psi = \frac{a\sinh(Ny)}{N^3\cosh(Nh)} - \frac{ay}{N^2}.$$

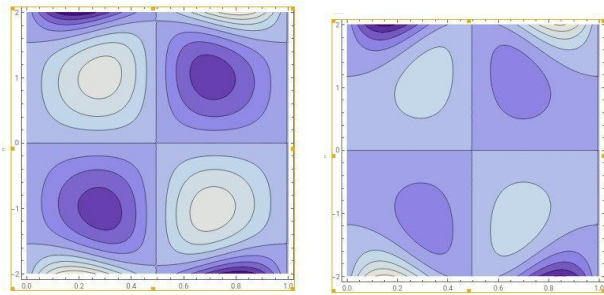
Graph



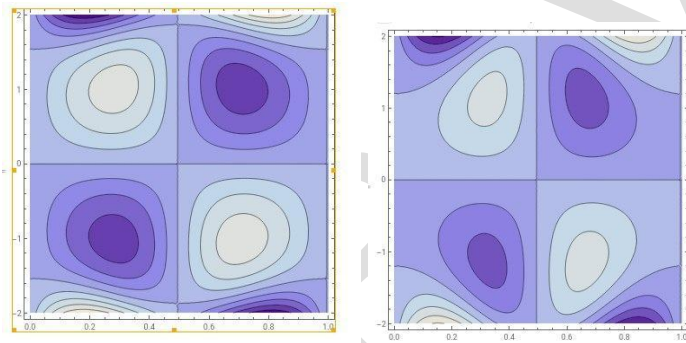
ϵ - Amplitude ratio
For $\epsilon = 0.1, 0.2, 0.001$



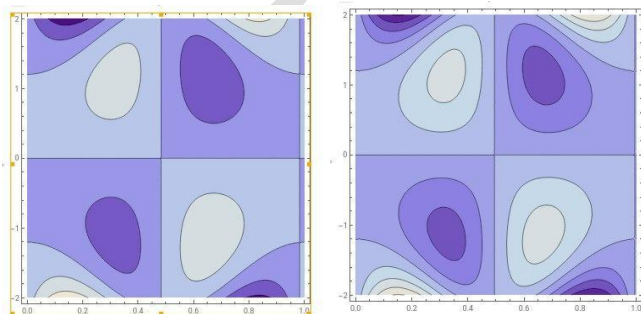
M- magnetic parameter
M=1,2



β – casson fluid parameter
 $\beta = 1, 2$



P= Damping force parameter
p=0.1,0.2,0.3 2,3



Result

We have plotted stream plot and contour plot to analyse the Casson fluid equation velocity profile.

The streamline (contour) plots illustrate the trapping phenomenon, where closed streamlines form bolus-like structures inside the flow field. The effect of the Casson parameter (β) and magnetic parameter (M) on trapping was studied.

It was observed that for smaller values of β , larger bolus regions are formed inside the channel, indicating stronger recirculation. As β increases, the size of the trapped bolus decreases, which means the non-Newtonian nature of the Casson fluid suppresses the trapping effect. This shows that higher yield stress in Casson fluid resists the circulation of fluid particles.

Similarly, the magnetic parameter M was found to have a significant influence. For low values of M , large bolus regions exist within the streamlines. However, as M increases, the bolus gradually reduces in size and may eventually disappear.

The velocity profiles for M values show a decreasing trend with an increase in the magnetic parameter. As M increases, the velocity magnitude reduces, indicating that the magnetic field exerts a resistive force (Lorentz force) which suppresses the motion of the fluid. The amplitude ratio exhibits the opposite behaviour. With an increase in ε , the velocity profiles shift upward and the magnitude of increases. This shows that the amplitude ratio enhances the oscillations and promotes higher fluid motion. Thus, it has an enhancing effect on the velocity distribution.

The effect of the damping parameter is similar to that of the magnetic parameter. As p increases, the velocity profiles are suppressed and the overall flow strength decreases. This indicates that the damping force opposes the motion and contributes to reducing oscillations in the system.

These observations confirm that both the Casson parameter and the magnetic parameter act to dampen bolus formation in peristaltic transport. This has important applications in controlling fluid recirculation in physiological and industrial processes.

EXTENDED LITERATURE REVIEW

Peristalsis is an important biological process which involves the compression and relaxation of muscles in a rhythmic pattern, forming a wavy channel to ease body fluid movement. This phenomenon has emerged out as a significant field of study due to its applicability in various dynamical systems. A non-Newtonian Fluid, Ellis's fluid is widely used to represent the peristaltic mechanism of biological systems. Prasad¹ et. al. studied how slip boundary conditions and magnetic fields affect peristaltic propulsion of Elli's fluid in a homogeneous channel, finding that increased slip parameter and magnetic field strength significantly influence the velocity profile. Kanwal² et. al. analyzed wavy channels with varying particle density under magnetic fields, revealing that both particle distribution and thermal conductivity variations can enhance or suppress heat transport. Waqar³ et. al. examined the thermo-physical behaviour of magnetized Ellis's fluid, observing that increasing the Ellis fluid and magnetic parameters decreased velocity in magnetized Elli's flow. Das⁴ et. al. analysed the heat transfer and entropy generation of Ellis Fluid through an inclined micro channel, finding the dual effect of augmenting Ellis's parameter on the increased velocity and on the Bejan number. Asghar⁵ et. al. provided a theoretical analysis of Ellis motion in non-uniform geometries. Understanding these effects is important for improving the utilization of a theoretical model in the real sense.

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